

# **Program Testing and Analysis:**

## **Path Profiling**

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# Warm-up Quiz

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What does the following code print?

```
function d(a, b) {  
  var sum = 0;  
  for (var i = 0; i < arguments.length; i++) {  
    sum += arguments[i];  
  }  
  console.log(sum == (a + b + 67));  
}
```

```
d(23, 45, 67);
```

true

false

Something else

# Warm-up Quiz

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}
```

```
d(23, 45, 67);
```

**Includes all passed arguments,  
independent of declared  
parameters**

**true**

false

Something else

# Outline

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- 1. Motivation and Challenges**
- 2. Ball-Larus algorithm for DAGs**
- 3. Generalization and Applications**

Mostly based on this paper:

- *Efficient path profiling*, Ball and Larus, MICRO 1996

Other reading material:

- *Whole program paths*, Larus, PLDI 1999
- *HOLMES: Effective statistical debugging via efficient path profiling*, Chilimbi et al., ICSE 2009

# Path Profiling

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- **Goal: Count how often a path through a function is executed**
- **Interesting for various applications**
  - Profile-directed **compiler optimizations**
  - **Performance tuning**: Which paths are worth optimizing?
  - **Test coverage**: Which paths are not yet tested?

# Challenges

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- **Runtime overhead**

- Limit slowdown of program

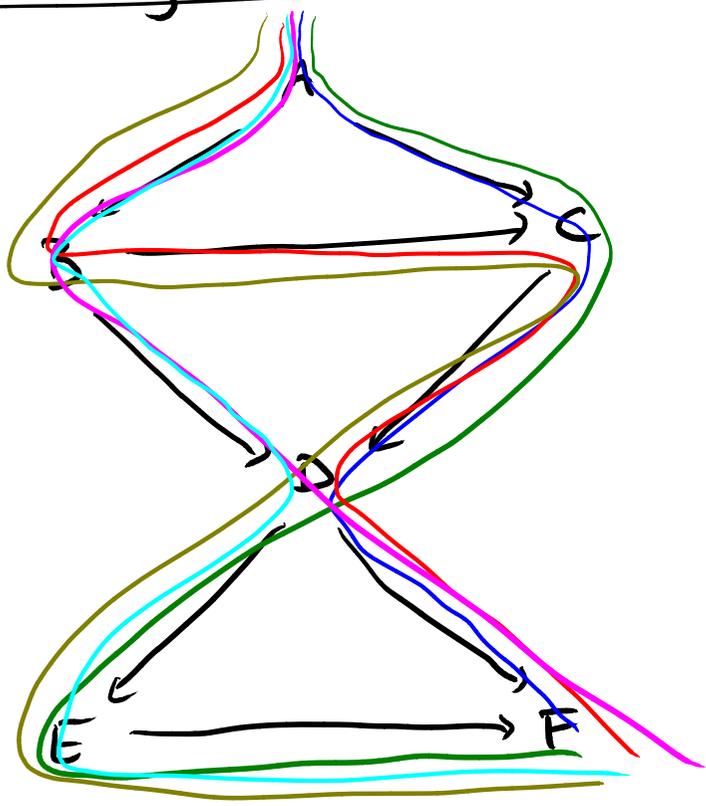
- **Accuracy**

- Ideally: **Precise profiles** (no heuristics, no approximations)

- **Infinitely many paths**

- Cycles in control flow graph

# Running example



#	Path	Frequency
0	ACDF	}
1	ACDEF	
2	ABCDF	
3	ABCDEF	
4	ABDF	
5	ABDEF	0

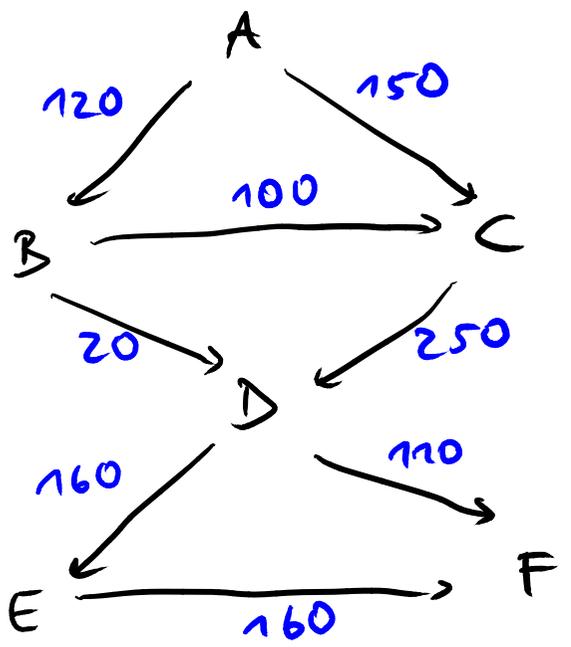
# Edge Profiling

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Naive approach: **Edge profiling**

- Instrument each branching point
- Count how often each CFG edge is executed
- Estimate **most frequent path**: Always follow most frequent edge

# Example: Edge profiling



frequency of execution

Quiz: What is the most frequent path?

ACDEF

Really? Two possible path profiles

Path	Profile 1	Profile 2
ACDF	90	110
ACDEF	60	40
ABCDF	0	0
ABCDEF	100	100
ABDF	20	0
ABDEF	0	20

# Edge Profiling

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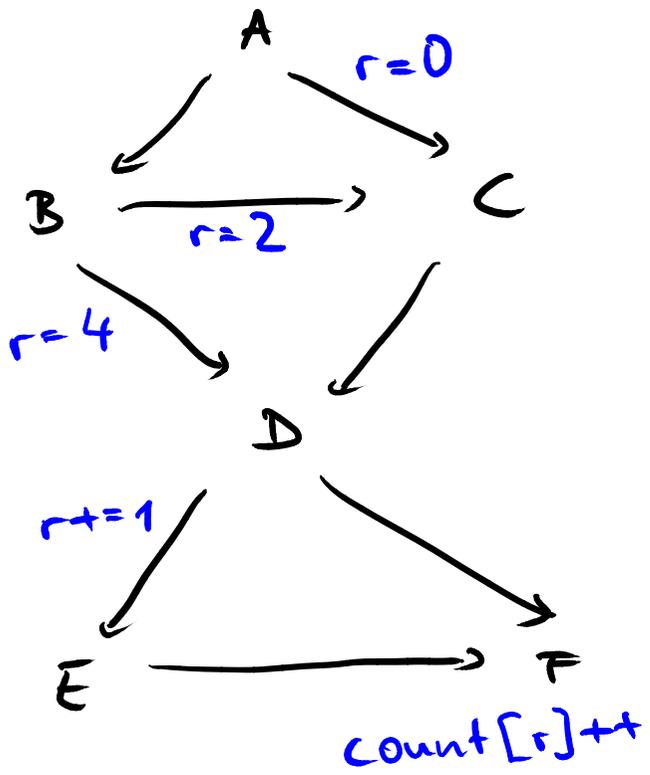
**Fails to uniquely identify most frequent path**

# Ball-Larus Algorithm

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- Assign a **number to each path**
- Compute path number by **incrementing a counter** at branching points
- **Properties of path encoding**
  - Precise: A single **unique encoding for each path**
  - Minimal: Instruments subset of edges with **minimal cost**

# Example: Path Encoding



instrumentation:

- state "r"
- array of counters "count"

Path	Encoding
ACDF	0
ACDEF	1
ABCDF	2
ABCDEF	3
ABDF	4
ABDEF	5

# Algorithm for DAGs

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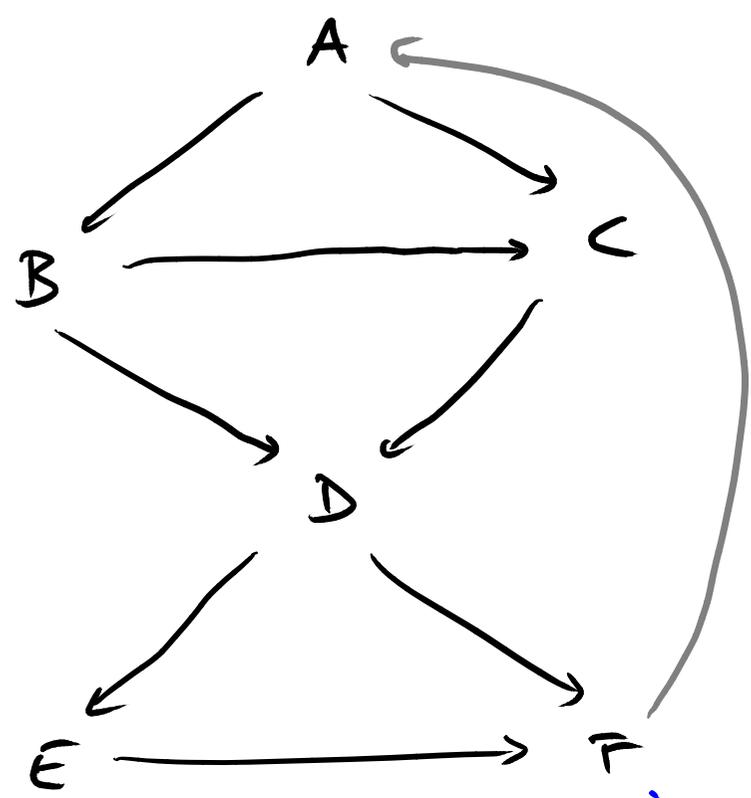
## Assumptions

- Control flow graph is a directed acyclic graph (**DAG**)
- $n$  paths (numbered 0 to  $n - 1$ )
- Graph has unique **entry and exit** nodes
- **Artificial back edge** from exit to entry

Assumptions

unique node

graph is acyclic



artificial backedge

unique exit node

# Algorithm: Overview

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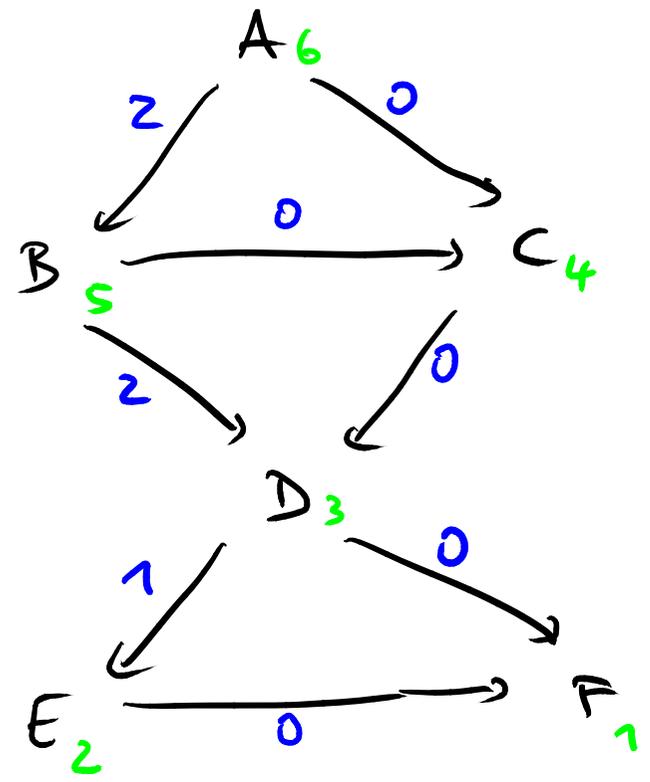
- **Step 1: Assign integers to edges**
  - Goal: Sum along a path yields unique number for path
  - Enough to achieve "precise" goal
- **Step 2: Assign increment operations to edges**
  - Goal: Minimize additions along edges
  - Instrument subset of all edges
  - Assumes to know/estimate how frequent edges are executed

# Representing Paths with Sums

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- Associate with each node a value:  
 $NumPaths(n)$  = number of paths from  $n$  to exit
- Computing  $NumPaths$ 
  - Visit nodes in reverse topological order
  - If  $n$  is leaf node:  
 $NumPaths(n) = 1$
  - Else:  
 $NumPaths(n) = \text{sum of } NumPaths \text{ of destination of outgoing edges}$

### Example: Num Paths



Values for edges

Reverse topological order:  
Successor of n visited before n

Node n	Num Paths(n)
F	1
E	1
D	2
C	2
B	4
A	6

# Representing Paths with Sums (2)

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**For each node in reverse topological order:**

■ **If  $n$  is leaf node:**

$$NumPaths(n) = 1$$

■ **Else:**

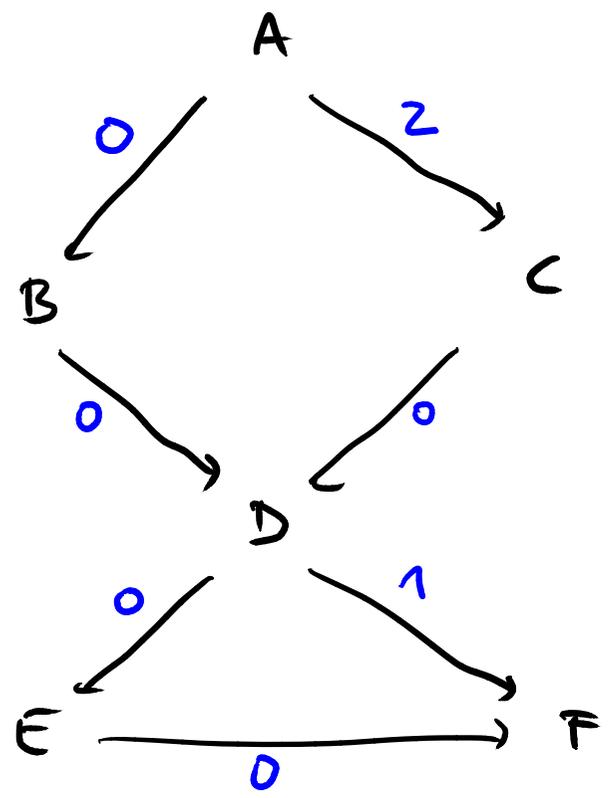
□  $NumPaths(n) = 0$

□ For each edge  $n \rightarrow m$ :

\*  $Val(n \rightarrow m) = NumPaths(n)$

\*  $NumPaths(n) += NumPaths(m)$

# Quiz: Values for Edges



n	NumPath (n)
A	4
B	2
C	2
D	2
E	1
F	1

- Sum of all NumPaths? 12
- Encoding for ACDF? 3

# Algorithm: Overview

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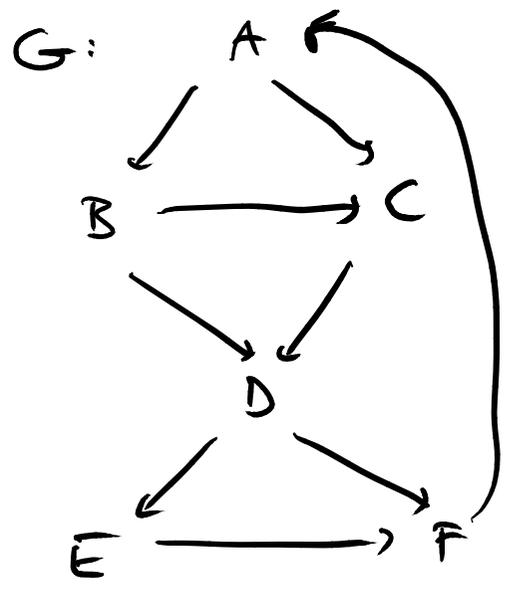
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# Spanning Tree

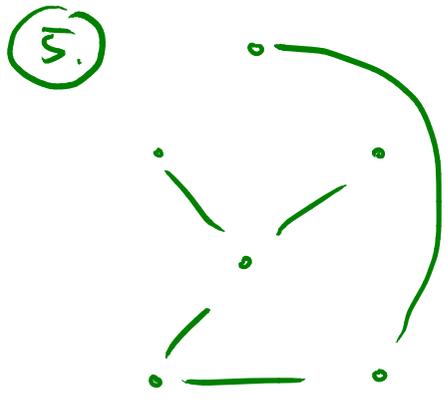
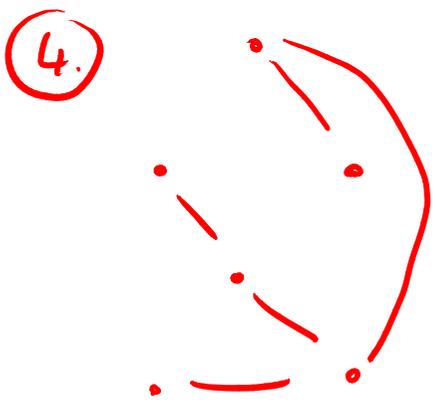
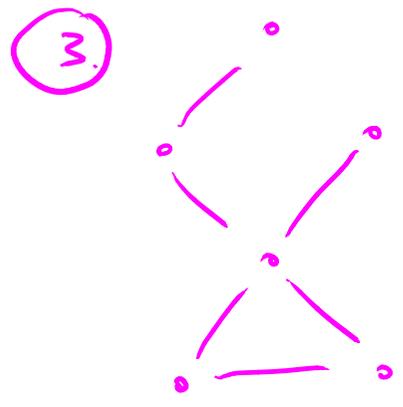
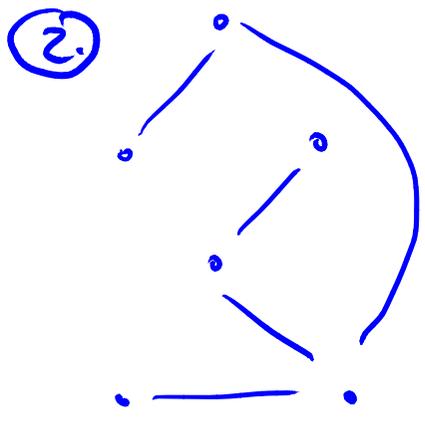
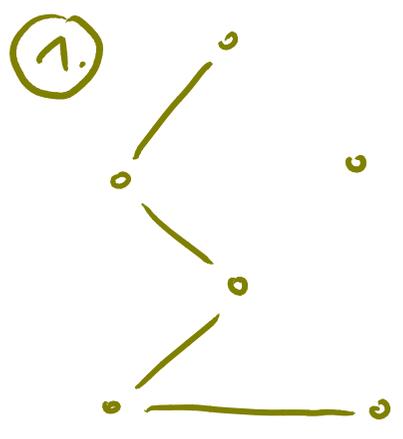
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- **Given: Graph  $G$**
- **Spanning tree  $T$ :**  
**Undirected** subgraph of  $G$  that is a **tree** and that contains **all nodes** of  $G$
- **Chord edges:** Edges in  $G$  but not in  $T$

# Example: Spanning Tree



Which of these is a spanning tree of G?



Solution:  
2, 4, 5

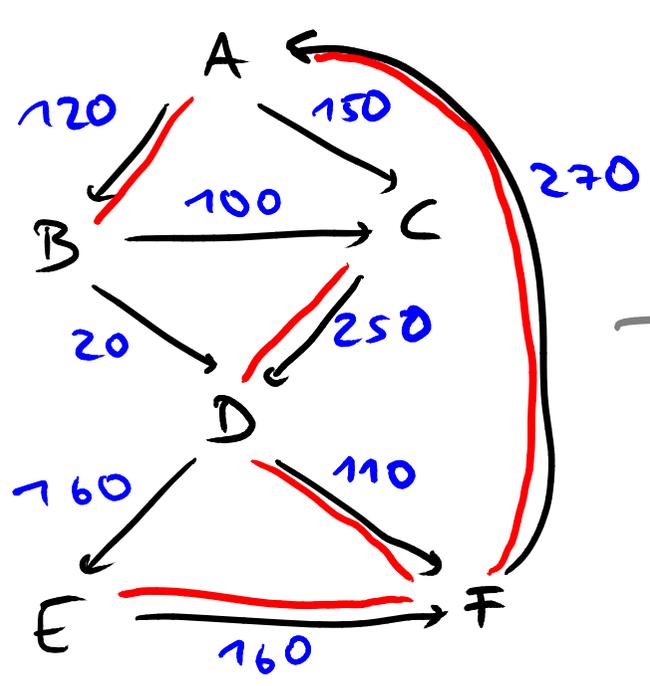
# Increments for Edges

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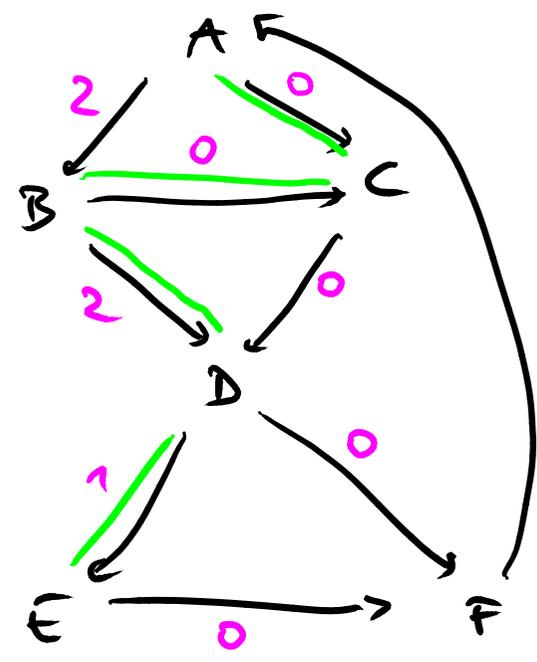
**Goal: Increment sum at subset of edges**

- **Choose spanning tree with maximum edge cost**
  - Cost of individual edges is assumed to be known
- **Compute increments at the chords of the spanning tree**
  - Based on existing event counting algorithm

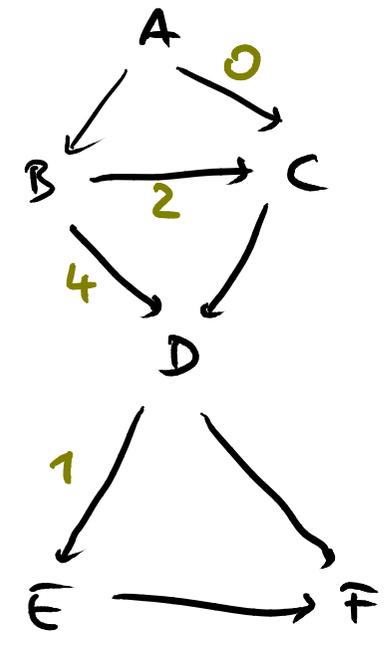
# Example: Increments for Edges



edge cost  
 most expensive  
 spanning tree



Chord edges  
 non-minimal  
 increments



minimal  
 increments  
 path encoding

# Instrumentation

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## ■ Basic idea

- Initialize sum at entry:  $r=0$
- Increment at edges:  $r+=. .$
- At exit, increment counter for path:  
`count [ r ] ++`

## ■ Optimization

- Initialize with incremented value, if first chord on path:  $r=. .$
- Increment sum and counter for path, if last chord on path: `count [ r+. . ] ++`

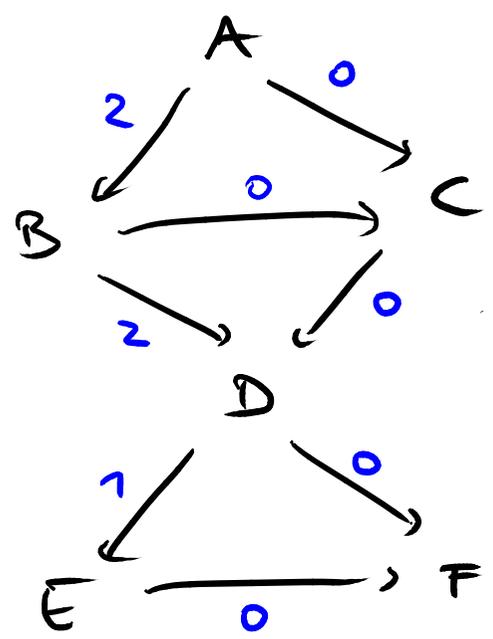
# Regenerating the Path

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Knowing the sum  $r$ , how to **determine the path?**

- Use edge values from step 1 (“non-minimal increments”)
- Start at entry with  $R = r$
- At branches, use edge with largest value  $v$  that is smaller than  $R$  and set  $R \leftarrow v$

Example: Regenerating the Path



$r = 4$ : A B D F

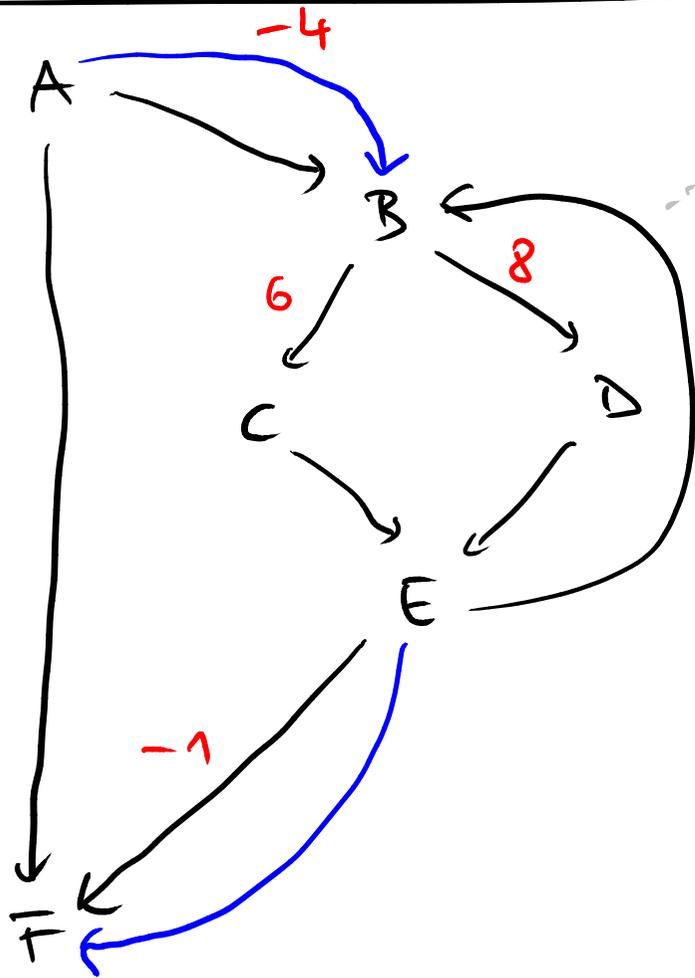
$r = 1$ : A C D E F

# Generalizing to Cyclic CGFs

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- For each backedge  $n \rightarrow m$ , add **dummy edges**
  - $Entry \rightarrow m$
  - $n \rightarrow Exit$
- **Remove backedges** and add **DAG-based increments**
- In addition, add **instrumentation to each backedge**
  - `count[r]++; r=0`

# Example: Generalizing



dummy edges

increments without  
backedge

backedge

Path	Encoding
AF	0
ABCEF	1
ABCE	2
⋮	
BCEF	5
BCE	6
⋮	
BDE	8

# Conclusions

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- **Ball-Larus profiling:**  
Beautiful algorithm with various applications
- Path profiles also determine accurate **edge and basic block profiles**
- Can **generalize to other graphs**, e.g., to compute call graph profiles