

# **Program Analysis**

## **Symbolic and Concolic Execution**

**Prof. Dr. Michael Pradel**

**Software Lab, University of Stuttgart**

**Winter 2023/2024**

# Warm-up Quiz

---

How many lines does this Python code print?

```
print ("\n")  
print (" \n")  
print (r" \n")  
print (r" \ \n")
```

4

5

6

None

# Warm-up Quiz

---

How many lines does this Python code print?

```
print ("\n")  
print ("\ \n")  
print (r" \ \n")  
print (r" \ \ \n")
```

4

5

6

None

# Warm-up Quiz

---

How many lines does this Python code print?

**Normal string: Backslash is an escape character**

```
print ("\n")  
print ("\ \n")  
print (r" \n")  
print (r" \\n")
```

**Raw string: Backslash is kept as-is**

4

5

6

None

# Warm-up Quiz

---

How many lines does this Python code print?

**Normal string: Backslash is an escape character**

```
print ("\n")
```

```
print ("\\n")
```

```
print (r"\\n")
```

```
print (r"\\\n")
```

**Raw string: Backslash is kept as-is**

**Output:**

\n

\\n

\\\n

4

5

6

None

# Overview

---

1. Classical **Symbolic Execution** ←
2. **Challenges** of Symbolic Execution
3. **Concolic** Testing
4. Large-Scale Application in **Practice**

Mostly based on these papers:

- *DART: directed automated random testing*, Godefroid et al., PLDI'05
- *KLEE: Unassisted and Automatic Generation of High-Coverage Tests for Complex Systems Programs*, Cadar et al., OSDI'08
- *Automated Whitebox Fuzz Testing*, Godefroid et al., NDSS'08

# Symbolic Execution

---

- Reason about behavior of program by "executing" it with **symbolic values**
- Originally proposed by James King (1976, CACM) and Lori Clarke (1976, IEEE TSE)
- Became **practical** around 2005 because of **advances in constraint solving** (SMT solvers)

# Example

---

```
function f(a, b, c) {  
  var x = y = z = 0;  
  if (a) {  
    x = -2;  
  }  
  if (b > 5) {  
    if (!a && c) {  
      y = 1;  
    }  
    z = 2;  
  }  
  assert(x + y + z != 3);  
}
```



## Concrete execution

$$a = b = c = 1$$

$$x = y = z = 0$$

true

$$x = -2$$

false

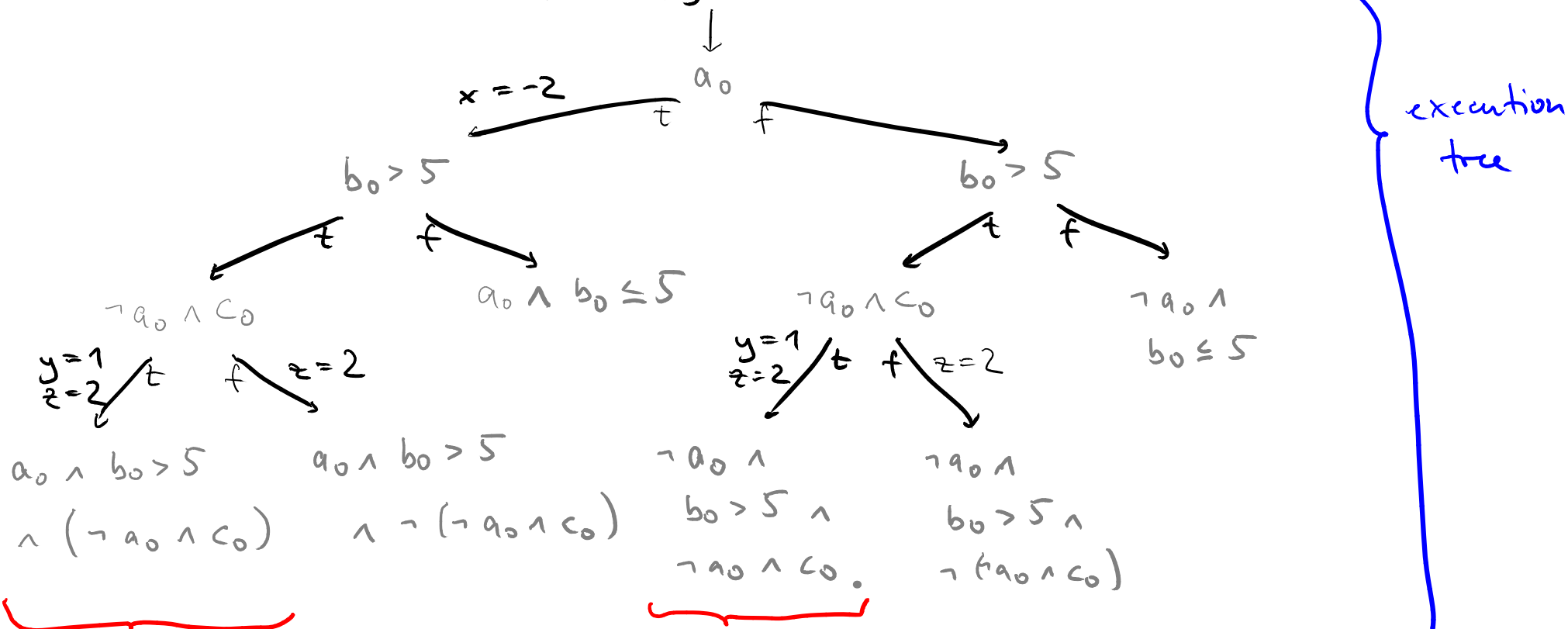
$$-2 + 0 + 0 \neq 3 \quad \checkmark$$

Symbolic execution

$a = a_0, b = b_0, c = c_0$

} symbolic values

$x = 0, y = 0, z = 0$



execution tree

$a_0 \wedge b_0 > 5 \wedge (\neg a_0 \wedge c_0)$

infeasible

$a_0 \wedge b_0 > 5 \wedge \neg(\neg a_0 \wedge c_0)$

$\neg a_0 \wedge b_0 > 5 \wedge \neg a_0 \wedge c_0$

$0 + 1 + 2 = 3$

↳ assertion violated

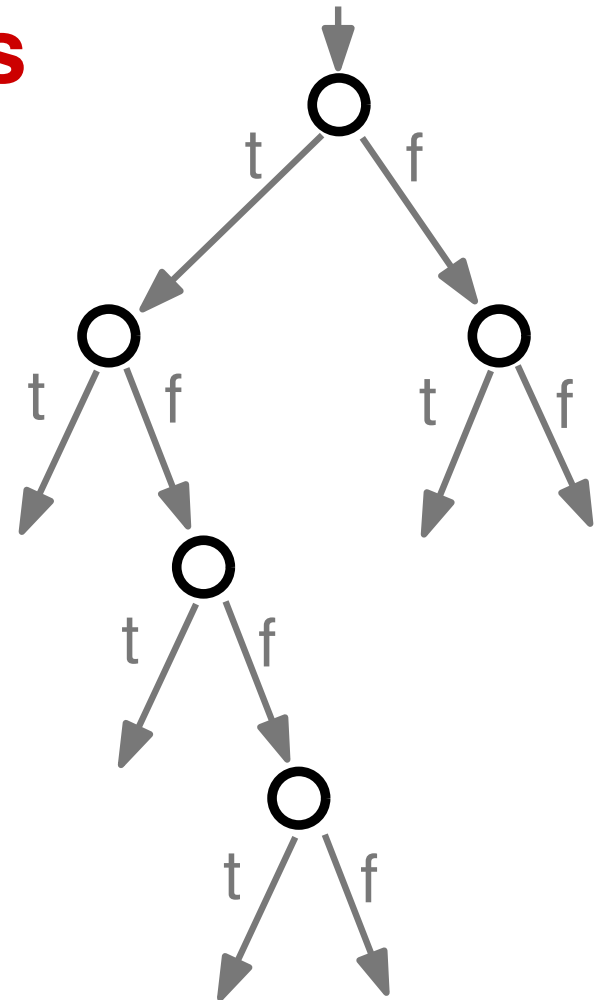
$\neg a_0 \wedge b_0 > 5 \wedge \neg(\neg a_0 \wedge c_0)$

# Execution Tree

---

## All possible execution paths

- Binary tree
- Nodes: **Conditional statements**
- Edges: Execution of sequence on non-conditional statements
- Each **path** in the tree represents an **equivalence class of inputs**



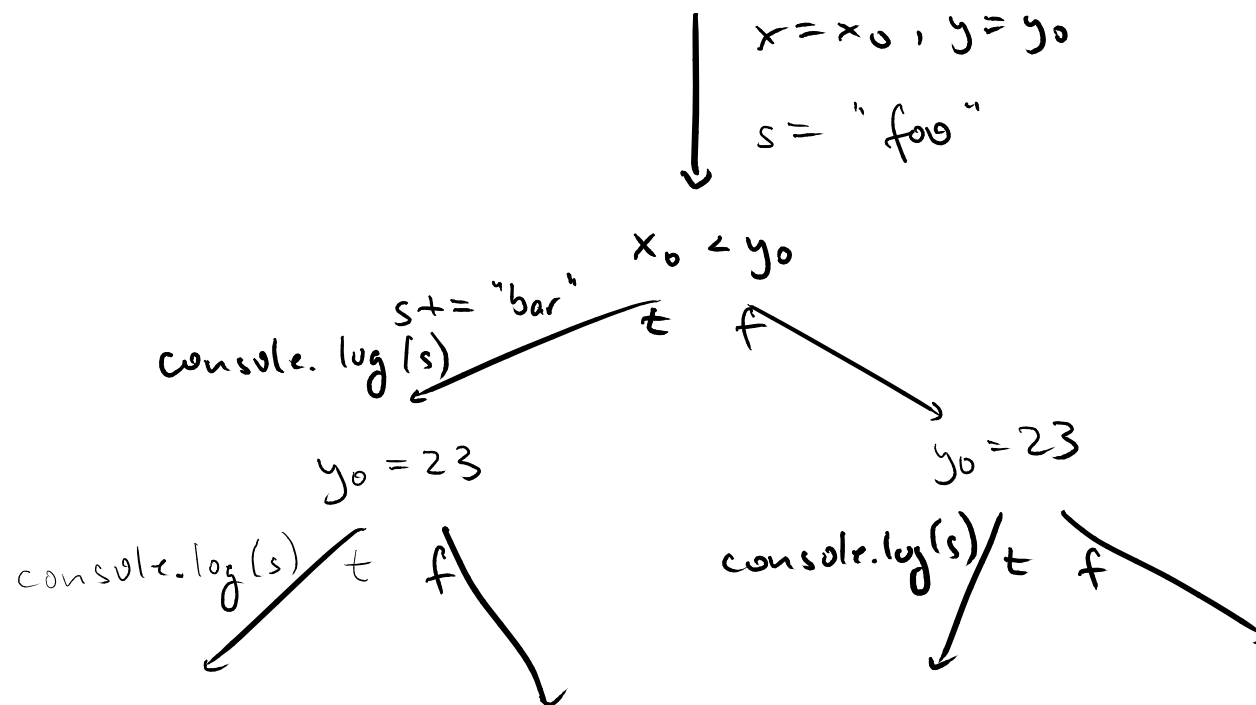
# Quiz

---

Draw the execution tree for this function. How many nodes and edges does it have?

```
function f(x,y) {  
  var s = "foo";  
  if (x < y) {  
    s += "bar";  
    console.log(s);  
  }  
  if (y === 23) {  
    console.log(s);  
  }  
}
```

# Quiz



→ 3 nodes  
(root & leaves not counted)

→ 7 edges

# Symbolic Values and Symbolic State

---

- **Unknown values**, e.g., user inputs, are kept symbolically
- **Symbolic state** maps variables to symbolic values

```
function f(x, y) {  
    var z = x + y;  
    if (z > 0) {  
        ...  
    }  
}
```

# Symbolic Values and Symbolic State

---

- **Unknown values**, e.g., user inputs, are kept symbolically
- **Symbolic state** maps variables to symbolic values

```
function f(x, y) {  
  var z = x + y;  
  if (z > 0) {  
    ...  
  }  
}
```

Symbolic input  
values:  $x_0, y_0$

Symbolic state:  
 $z = x_0 + y_0$

# Path Conditions

---

**Quantifier-free formula** over the symbolic inputs that encodes all **branch decisions** taken so far

```
function f(x, y) {  
    var z = x + y;  
    if (z > 0) {  
        ...  
    }  
}
```



# Path Conditions

---

**Quantifier-free formula** over the symbolic inputs that encodes all **branch decisions** taken so far

```
function f(x, y) {  
    var z = x + y;  
    if (z > 0) {  
        ...  
    }  
}
```

Path condition:

$$x_0 + y_0 > 0$$

# Satisfiability of Formulas

---

Determine whether a path is **feasible**:

Check if its path condition is satisfiable

- Done by powerful **SMT/SAT solvers**
  - SAT = satisfiability,  
SMT = satisfiability modulo theory
  - E.g., Z3, Yices, STP
- For a satisfiable formula, solvers also provide a **concrete solution**
- Examples:
  - $a_0 + b_0 > 1$ : Satisfiable, one solution:  $a_0 = 1, b_0 = 1$
  - $(a_0 + b_0 < 0) \wedge (a_0 - 1 > 5) \wedge (b_0 > 0)$ : Unsatisfiable

# Applications of Symbolic Execution

---

- General goal: Reason about behavior of program
- Basic applications
  - Detect infeasible paths
  - Generate test inputs
  - Find bugs and vulnerabilities
- Advanced applications
  - Generating program invariants
  - Prove that two pieces of code are equivalent
  - Debugging
  - Automated program repair

# Overview

---

1. Classical **Symbolic Execution**
2. **Challenges** of Symbolic Execution ←
3. **Concolic** Testing
4. Large-Scale Application in **Practice**

Mostly based on these papers:

- *DART: directed automated random testing*, Godefroid et al., PLDI'05
- *KLEE: Unassisted and Automatic Generation of High-Coverage Tests for Complex Systems Programs*, Cadar et al., OSDI'08
- *Automated Whitebox Fuzz Testing*, Godefroid et al., NDSS'08

# Problems of Symbolic Execution

---

- **Loops and recursion**: Infinite execution trees
- **Path explosion**: Number of paths is exponential in the number of conditionals
- **Environment modeling**: Dealing with native/system/library calls
- **Solver limitations**: Dealing with complex path conditions
- **Heap modeling**: Symbolic representation of data structures and pointers

# Problems of Symbolic Execution

---

- **Loops and recursion:** Infinite execution trees
- **Path explosion:** Number of paths is exponential in the number of conditionals
- **Environment modeling:** Dealing with native/system/library calls
- **Solver limitations:** Dealing with complex path conditions
- **Heap modeling:** Symbolic representation of data structures and pointers

## Path Explosion

```
function f(a) {
```

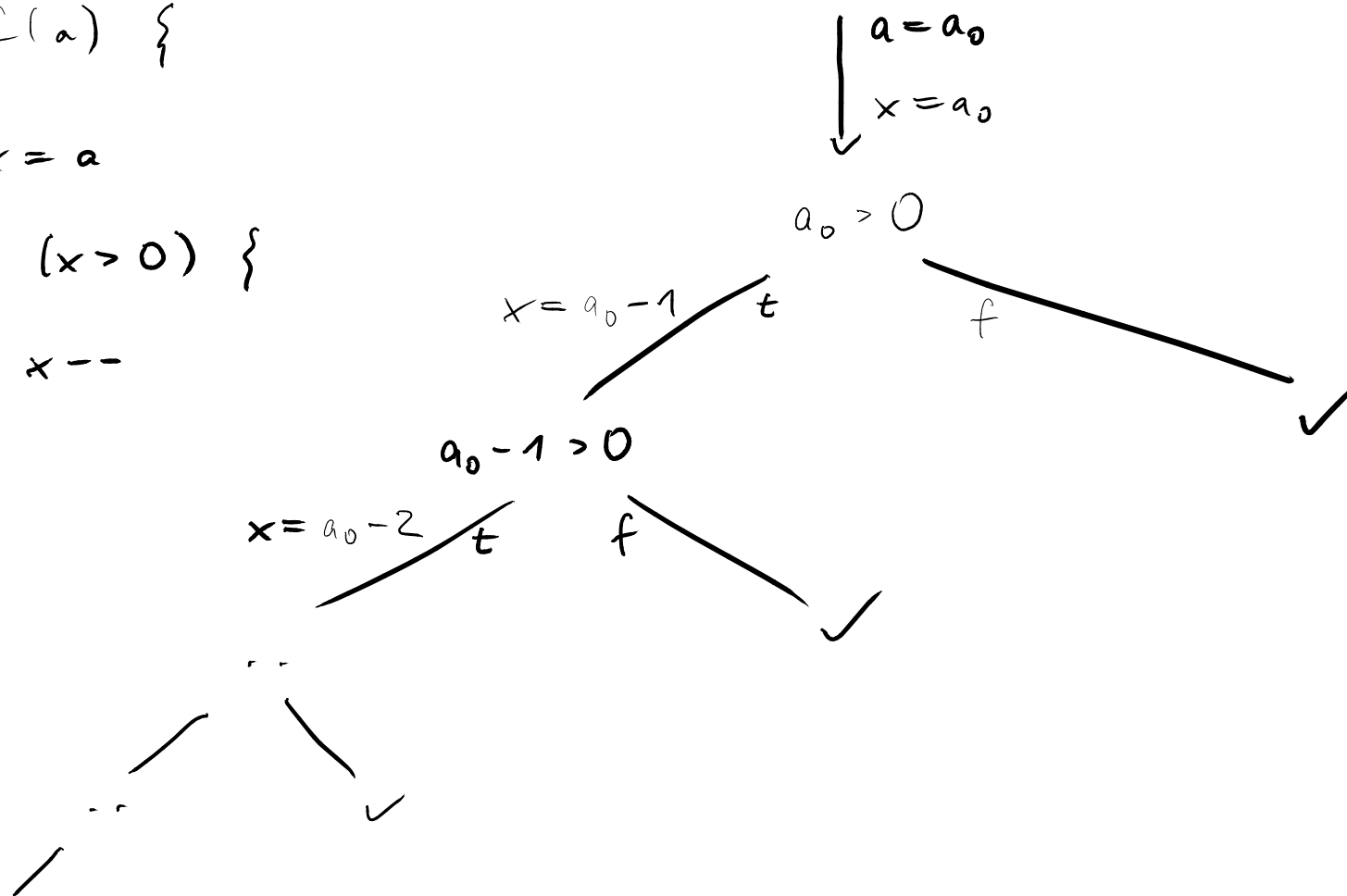
```
  var x = a
```

```
  while (x > 0) {
```

```
    x--
```

```
  }
```

```
}
```

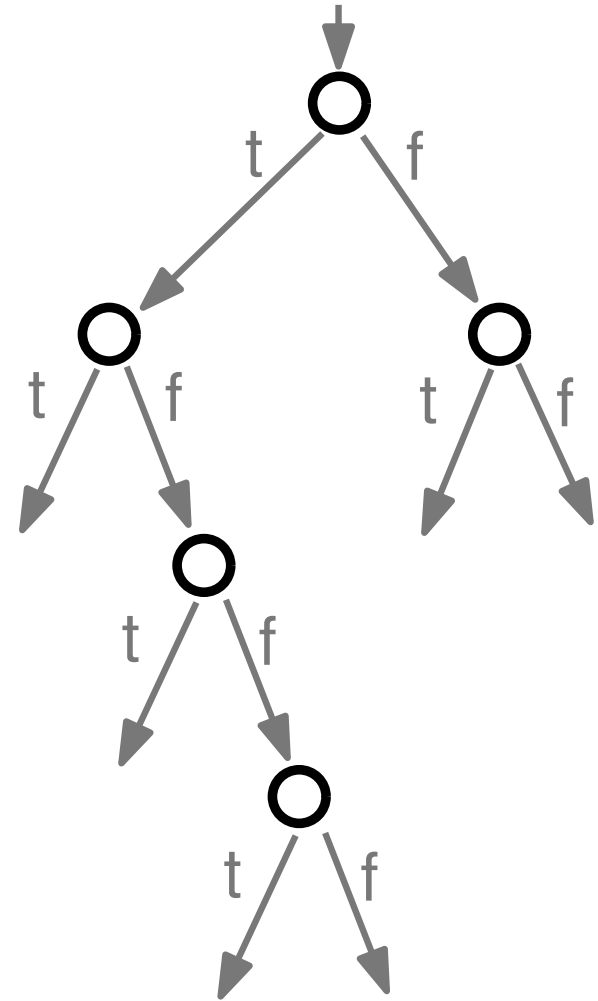


# Dealing with Large Execution Trees

---

**Heuristically** select which branch to explore next

- Select at **random**
- Select based on **coverage**
- Prioritize based on distance to **"interesting"** program locations
- **Interleaving** symbolic execution with **random testing**





# Problems of Symbolic Execution

---

- **Loops and recursion:** Infinite execution trees
- **Path explosion:** Number of paths is exponential in the number of conditionals
- **Environment modeling:** Dealing with native/system/library calls
- **Solver limitations:** Dealing with complex path conditions
- **Heap modeling:** Symbolic representation of data structures and pointers

# Problems of Symbolic Execution

---

- **Loops and recursion**: Infinite execution trees
- **Path explosion**: Number of paths is exponential in the number of conditionals
- **Environment modeling**: Dealing with native/system/library calls
- **Solver limitations**: Dealing with complex path conditions
- **Heap modeling**: Symbolic representation of data structures and pointers

# Modeling the Environment

---

- Program behavior may depend on **parts of system not analyzed** by symbolic execution
- E.g., native APIs, interaction with network, file system accesses

```
var fs = require("fs");  
var content = fs.readFileSync("/tmp/foo.txt");  
if (content === "bar") {  
    ...  
}
```

# Modeling the Environment (2)

---

## Solution implemented by **KLEE**

- If all arguments are concrete, forward to OS
- Otherwise, provide **models that can handle symbolic files**
  - Goal: Explore all possible legal interactions with the environment

```
var fs = {  
  readFileSync: function(file) {  
    // doesn't read actual file system, but  
    // models its effects for symbolic file names  
  }  
}
```

# Problems of Symbolic Execution

---

- **Loops and recursion:** Infinite execution trees
- **Path explosion:** Number of paths is exponential in the number of conditionals
- **Environment modeling:** Dealing with native/system/library calls
- **Solver limitations:** Dealing with complex path conditions
- **Heap modeling:** Symbolic representation of data structures and pointers

# Problems of Symbolic Execution

---

- **Loops and recursion**: Infinite execution trees
- **Path explosion**: Number of paths is exponential in the number of conditionals
- **Environment modeling**: Dealing with native/system/library calls
- **Solver limitations**: Dealing with complex path conditions
- **Heap modeling**: Symbolic representation of data structures and pointers

**One approach: Mix symbolic with concrete execution**

# Overview

---

1. Classical **Symbolic Execution**
2. **Challenges** of Symbolic Execution
3. **Concolic Testing** ←
4. Large-Scale Application in **Practice**

Mostly based on these papers:

- *DART: directed automated random testing*, Godefroid et al., PLDI'05
- *KLEE: Unassisted and Automatic Generation of High-Coverage Tests for Complex Systems Programs*, Cadar et al., OSDI'08
- *Automated Whitebox Fuzz Testing*, Godefroid et al., NDSS'08

# Concolic Testing

---

**Mix concrete and symbolic execution =  
"concolic"**

- Perform concrete and symbolic execution side-by-side
- Gather path constraints while program executes
- After one execution, negate one decision, and re-execute with new input that triggers another path



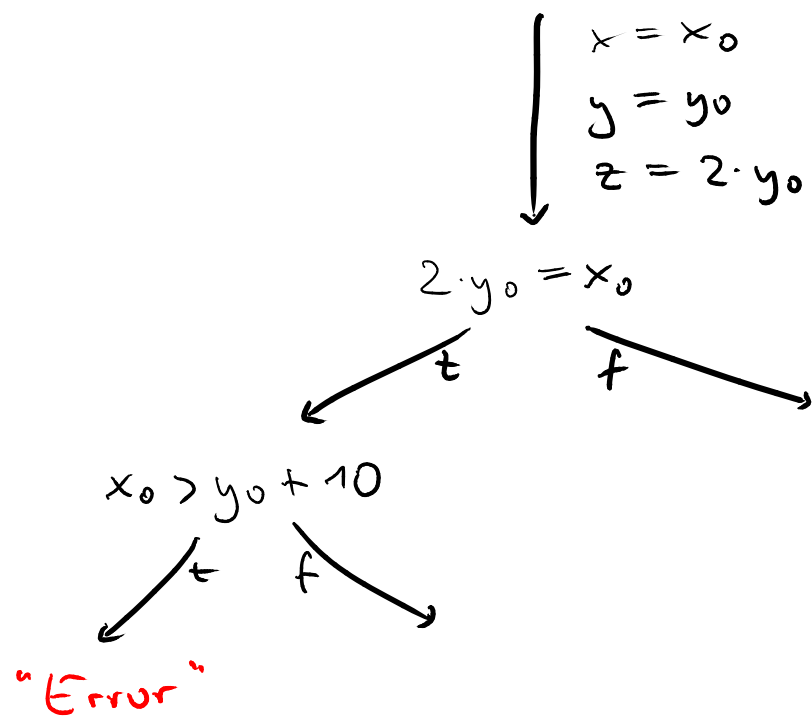
# Example

---

```
function double(n) {  
    return 2 * n;  
}
```

```
function testMe(x, y) {  
    var z = double(y);  
    if (z === x) {  
        if (x > y + 10) {  
            throw "Error";  
        }  
    }  
}
```

## Concolic execution: Execution tree



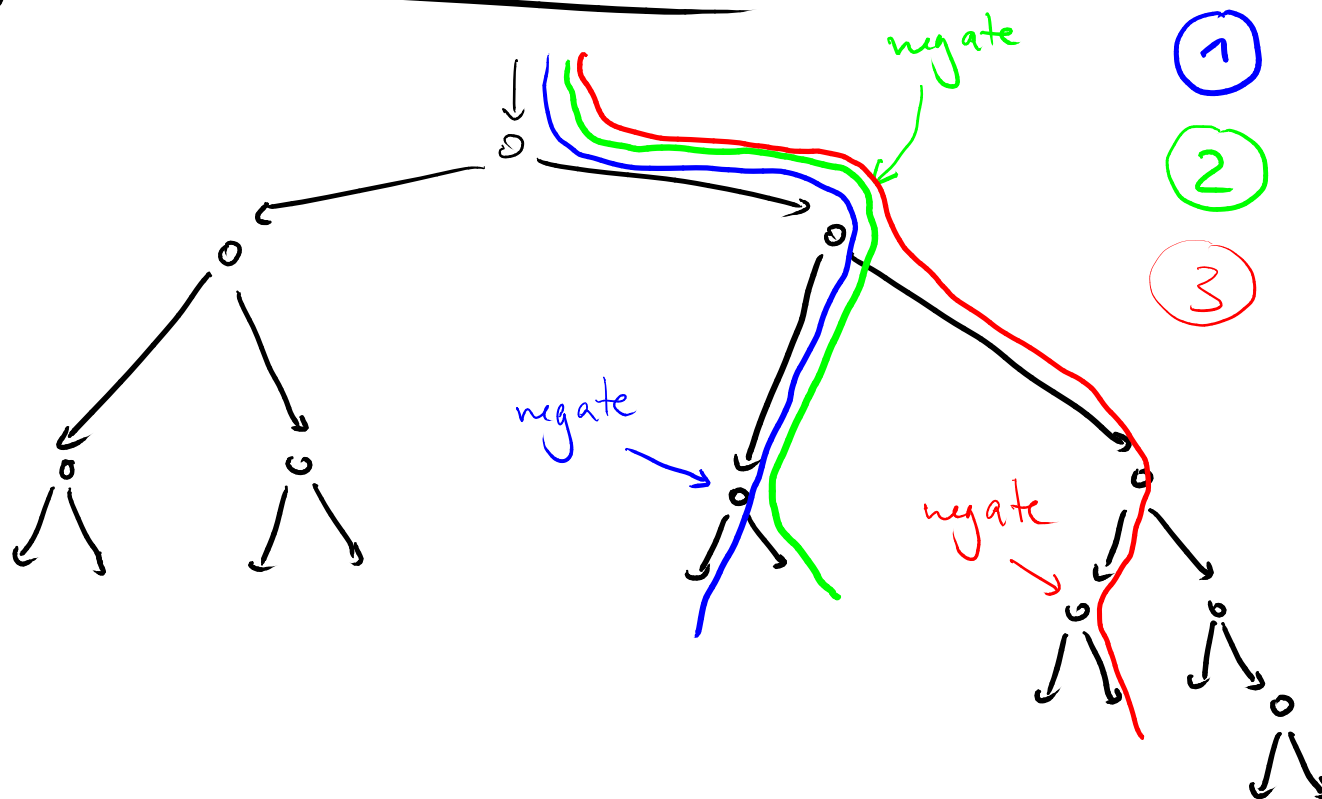
## Execution 1

Concrete execution	Symbolic exec.	Path conditions
$x = 22, y = 7$ (After entering the fct.)	$x = x_0, y = y_0$	
$x = 22, y = 7$ $z = 14$ (After call to double & assignment)	$x = x_0, y = y_0$ $z = 2 \cdot y_0$	
$x = 22, y = 7$ $z = 14$ (After outer if)	$x = x_0, y = y_0,$ $z = 2 \cdot y_0$	$2 \cdot y_0 \neq x_0$
	Solve: $2 \cdot y_0 = x_0$	
	Solutions: $x_0 = 2, y_0 = 1$	

## Execution 2

Concrete exec.	Symb. exec.	Path condition
$x=2, y=1$	$x=x_0, y=y_0$	
$x=2, y=1$ $z=2$	$x=x_0, y=y_0,$ $z=2 \cdot y_0$	
-u-	-"-	$2 \cdot y_0 = x_0$
-"-	-"-	$2 \cdot y_0 = x_0 \wedge$ $x_0 \leq y_0 + 10$
Solve: $2 \cdot y_0 = x_0 \wedge x_0 > y_0 + 10$		
Solution: $x_0 = 30, y_0 = 15$	} Hits "Error"	

# Exploring the Execution Tree



# Algorithm

---

## Repeat until all paths are covered

- **Execute** program with concrete input  $i$  and collect **symbolic constraints** at branch points:  $C$
- **Negate one constraint** to force taking an alternative branch  $b'$ : Constraints  $C'$
- Call constraint solver to **find solution** for  $C'$ : **New concrete input**  $i'$
- **Execute** with  $i'$  to take branch  $b'$
- Check at runtime that  $b'$  is indeed taken  
Otherwise: "divergent execution"

## Divergent Execution: Example

```
function f(a) {
```

```
  if (Math.random() < 0.5) {
```

```
    if (a > 1) {
```

```
      console.log("yes")
```

```
    }
```

```
  }
```

```
}
```

Exec. 1

$a = 0$

true

false

path constraint:

$a_0 \leq 1$

negate & solve:

$a_0 = 2$

Exec. 2

$a = 2$

false

↓

Divergent  
execution

# Benefits of Concolic Approach

---

When symbolic reasoning is impossible or impractical, **fall back to concrete values**

- Native/system/API functions
- Operations not handled by solver (e.g., floating point operations)



# Overview

---

1. Classical **Symbolic Execution**
2. **Challenges** of Symbolic Execution
3. **Concolic** Testing
4. Large-Scale Application in **Practice** ←

Mostly based on these papers:

- *DART: directed automated random testing*, Godefroid et al., PLDI'05
- *KLEE: Unassisted and Automatic Generation of High-Coverage Tests for Complex Systems Programs*, Cadar et al., OSDI'08
- *Automated Whitebox Fuzz Testing*, Godefroid et al., NDSS'08

# Large-Scale Concolic Testing

---

- **SAGE**: Concolic testing tool developed at Microsoft Research
- Test robustness against unexpected **inputs read from files**, e.g.,
  - Audio files read by media player
  - Office documents read by MS Office
- Start with known input files and handle **bytes read from files as symbolic input**
- Use concolic execution to compute variants of these files

# Large-Scale Concolic Testing (2)

---

- Applied to hundreds of applications
- Over **400 machine years of computation** from 2007 to 2012
- Found **hundreds of bugs**, including many security vulnerabilities
  - One third of all the bugs discovered by file fuzzing during the development of Microsoft's Windows 7

# Summary: Symbolic & Concolic Testing

---

## Solver-supported, whitebox testing

- Reason **symbolically** about (parts of) inputs
- Create new inputs that **cover not yet explored paths**
- More **systematic** but also more **expensive** than random and fuzz testing
- **Open challenges**
  - Effective exploration of huge search space
  - Other applications of constraint-based program analysis, e.g., debugging and automated program repair