Programming Paradigms

Type Systems (Part 2)

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Summer 2023

Overview

- **Introduction**
- **Types in Programming Languages**
- **Type Equivalence**
- **Type Compatibility and Conversions** ←
- **Formally Defined Type Systems**
  - Arithmetic Expressions
  - Lambda Calculus

Type Compatibility

- **Check whether combining two values is valid according to their types**
- **“Combining” may mean**
  - Assignment: Are left-hand side and right-hand side compatible?
  - Operators: Are operands compatible with the operator and with each other?
  - Function calls: Are actual arguments and formal parameters compatible?

Compatible $\neq$ Equal

Most PLs: Types may be compatible even when not the same

Example (C):

```
double d = 2.3;
float f = d * 2;
int i = f;
printf("%d\n", i);
```

Compatible $\neq$ Equal (2)

- **Rules of PL define which types are compatible**
- **Examples of rules**
  - Can assign subtype to supertype
  - Different number types are compatible with each other
  - Collections of same type are compatible, even if length differs

Type Conversions

When types aren't equal, they must be converted

- Option 1: Cast = explicit type conversion
  - Programmer changes value's type from T1 to T2
- Option 2: Coercion = implicit type conversion
  - PL allows values of type T1 in situation where type T2 expected

Runtime Behavior of Conversions

Three cases:

- Types are structurally equivalent:

  Conversion is only conceptual, no code generated
- Types have different sets of values, but are represented in the same way in memory:

  May need check that value is in target type
- Different low-level representations:

  Need special instructions for conversion

Examples (Ada)

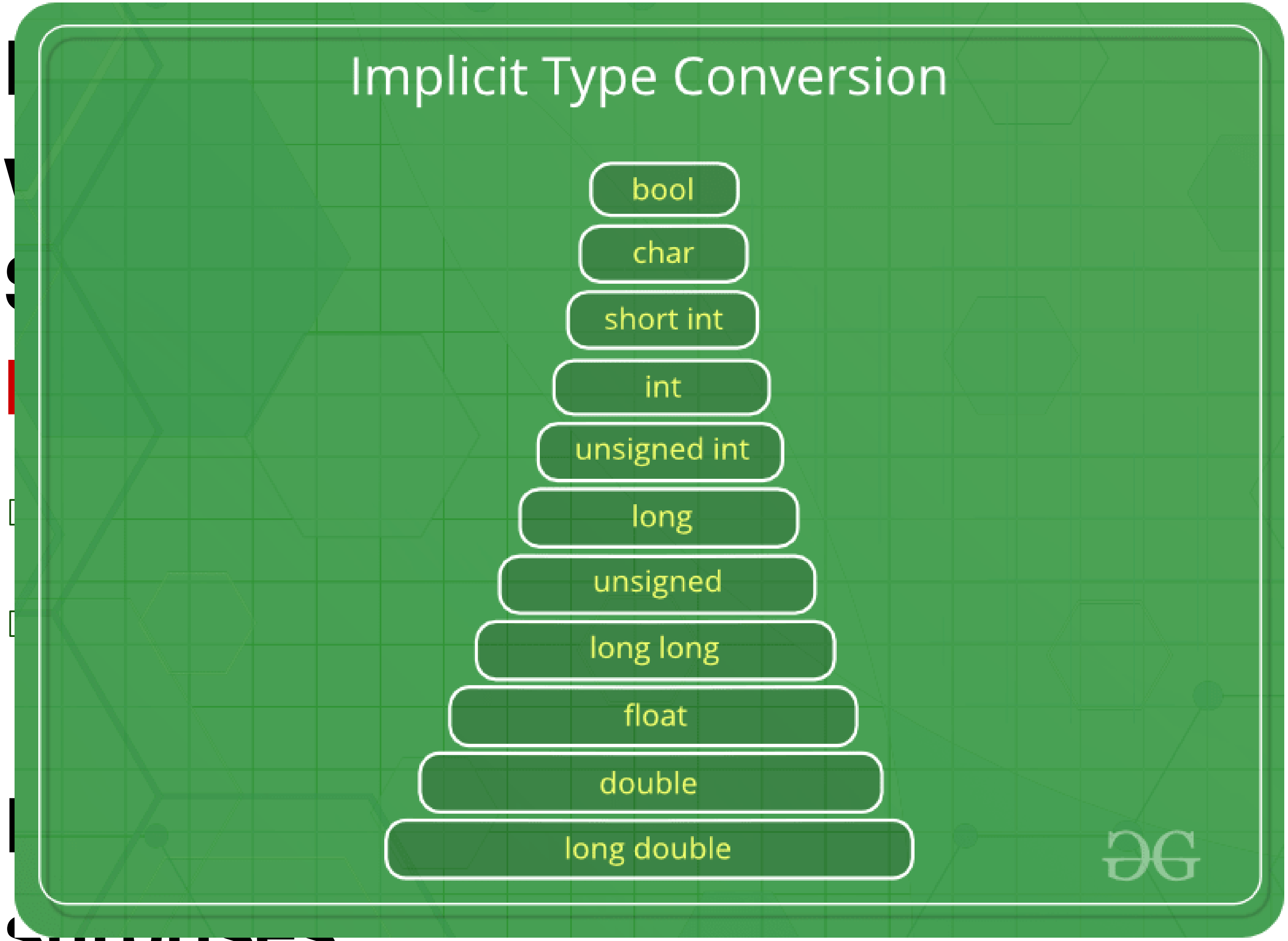
```
n: integer
r: long-float
t: test_score              -- integer range 0..100
c: celsius_temp            -- type alias for integer
t := test_score (n)        -- runtime, semantic check: in range 0..100?
n := integer (t)           -- no check needed
r := long-float (n)        -- runtime conversion
n := integer (r)           -- runtime conversion & check

n := integer (c)           -- purely conceptual
c := celsius_temp(n)       -- purely conceptual
```

Coercions in C

- **Most primitive types are coerced whenever needed**
- **Some coercions cause information loss**
  - `float` to `int`: Loose fraction
  - `int` to `char`: Causes `char` to overflow (and will give unexpected result)
- **Enable compiler warnings to avoid surprises**

Coercions in C

- I
  V
- S
- I
  surprises



Source: *geeksforgeeks.org*

Coercions in JavaScript

- **Almost all types are coerced when needed**
  - Rationale: Websites shouldn't crash
- **Some coercions make sense:**
  - `"number:" + 3` yields `"number:3"`
- **Many others are far from intuitive:**
  - `[1, 2] << "2"` yields `0`

More details and examples:
***The Good, the Bad, and the Ugly: An Empirical Study of Implicit Type Conversions in JavaScript*. Pradel and Sen. ECOOP 2015**

Quiz: Coercions in C

What does the following C code print?

```
float d = 1027.23;
int l = d;
d = l;
char c = d;
bool b = c;

printf("d=%f, ", d);
printf("l=%d, ", l);
printf("c=%d, ", c);
printf("b=%d\n", b);
```

Quiz: Coercions in C

What does the following C code print?

```
float d = 1027.23;
int l = d;      // coercion to integer 1027
d = l;          // fraction lost, d is 1027.0
char c = d;     // doesn’t fit; coerced to 3
bool b = c;     // coercion to true

printf("d=%f, ", d);   // 1027.00000
printf("l=%d, ", l);   // 1027
printf("c=%d, ", c);   // 3
printf("b=%d\n", b);   // 1
```

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Formally Defined Type Systems

- **Type systems are**
  - implemented in a compiler
  - formally described
  - and sometimes both
- **Active research area with dozens of papers each year**
  - Focus: New languages and strong type guarantees

Typed Expression: Syntax

t ::= true |
false |
if t then t else t |
0 |
succ t |
pred t |
iszero t

(semantics: not formally defined)

Examples:

succ 0 (= 1)

if (iszero (pred (succ (0))))
then 0
else (succ 0) (= 0)

Not All Expressions Make Sense

- **Only some expressions can be evaluated**
- **Others don’t make sense**
  - Implementation of the language would get stuck or throw a runtime error

Examples

if (iszero 0) then true else 0

succ (if 0 then true else (pred false))

expressions that don't make sense

if true then false else true : Bool

pred (succ (succ 0)) : Nat

Types to the Rescue

- **Use types to check whether an expression is meaningful**
  - If term t has a type T, then its evaluation won't get stuck
  - Written as $t : T$

”has type”

- **Two types**
  - Nat .. natural numbers
  - $Bool$ .. Boolean values

Type Rules

Background: $\frac{A}{B}$ .. rule

if A is true, then B is true

$\frac{}{B}$ .. axiom

B is always true

Bool:

$$\frac{}{\text{true} : \text{Bool}} \quad (\text{T-True})$$

$$\frac{}{\text{false} : \text{Bool}} \quad (\text{T-False})$$

$$\frac{t_1 : \text{Bool} \qquad t_2 : T \qquad t_3 : T}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 : T} \quad (\text{T-If})$$

Nat:

$$\frac{}{0 : \text{Nat}} \quad (\text{T-Zero})$$

$$\frac{t_1 : \text{Nat}}{\text{succ } t_1 : \text{Nat}} \quad (\text{T-Succ})$$

$$\frac{t_1 : \text{Nat}}{\text{pred } t_1 : \text{Nat}} \quad (\text{T-Pred})$$

$$\frac{t_1 : \text{Nat}}{\text{iszero } t_1 : \text{Bool}} \quad (\text{T-IsZero})$$

Type Checking Expressions

- **Typing relation**: Smallest binary relation between terms and types that satisfies all instances of the rules
- Term t is **typeable (or well typed)** if there is some T such that $t : T$
- **Type derivation**: Tree of instances of the typing rules that shows $t : T$

Type Derivation : Example 1

$$\dfrac{\dfrac{}{\text{true} : \text{Bool}}\,(\text{T-True}) \qquad \dfrac{}{\text{false} : \text{Bool}}\,(\text{T-False}) \qquad \dfrac{}{\text{true} : \text{Bool}}\,(\text{T-True})}{\text{if true then false else true} : \text{Bool}}\,(\text{T-If})$$

✓

Example 2

Can't apply any axiom or rule.

Expression is **not** well-typed!

$$\dfrac{\dfrac{\dfrac{??}{0 : \text{Bool}} \qquad \dfrac{??}{\text{true} : \text{Nat}} \qquad \dfrac{\ldots}{(\text{pred false}) : \text{Nat}}}{\text{if } 0 \text{ then true else } (\text{pred false}) : \text{Nat}}\ (\text{T-If})}{\text{succ } (\text{if } 0 \text{ then true else } (\text{pred false})) : \text{Nat}}\ (\text{T-Succ})$$

Quiz: Typing Derivation

Find the typing derivation for the following expression:

```
if false then (succ(pred 0)) else (succ 0)
```

How many axioms and rules do you need to apply?

→ 4 rules , 3 rules

$$\dfrac{\dfrac{}{\text{false} : \text{Bool}}\,(\text{T-False}) \quad \dfrac{\dfrac{\dfrac{}{0 : \text{Nat}}\,(\text{T-Zero})}{\text{pred } 0 : \text{Nat}}\,(\text{T-Pred})}{(\text{succ}(\text{pred } 0)) : \text{Nat}}\,(\text{T-Succ}) \quad \dfrac{\dfrac{}{0 : \text{Nat}}\,(\text{T-Zero})}{(\text{succ } 0) : \text{Nat}}\,(\text{T-Succ})}{\text{if false then (succ (pred 0)) else (succ 0)} : \text{Nat}}\,(\text{T-If})$$

Another Example

Try to find a typing derivation for the following expression:

```
if true then true else 0
```

No way of applying axioms/rules
so that both terms have the same type.

$$\dfrac{\dfrac{}{\text{true} : \text{Bool}}\;(\text{T-True}) \qquad \dfrac{}{\text{true} : ?} \qquad \dfrac{}{0 : ?}}{\text{if true then true else } 0 : ?}\;(\text{T-If})$$