Programming Paradigms

Syntax (Part 3)

Prof. Dr. Michael Pradel

Software Lab, University of Stuttgart

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Overview

- **Specifying syntax**
  - Regular expressions
  - Context-free grammars
- **Scanning**
- **Parsing** 
  - Top-down parsing
  - Bottom-up parsing

FOLLOW Sets

FOLLOW(A): Set of all terminals that may follow A in some derivation

- Including special symbol EOF for "end of file"
- Never includes ϵ

Example:

S $\rightarrow$ a B c	**FOLLOW(S) = { EOF }**
B $\rightarrow$ d	**FOLLOW(B) = { c }**

Computing FOLLOW Sets

To compute FOLLOW(A), apply these rules until all FOLLOW sets constant

- If A is start symbol, put EOF in FOLLOW(A)
- Productions of the form B $\rightarrow \alpha$ A β:

  Add FIRST(β) - $\{ \epsilon \}$ to FOLLOW(A)
- Productions of the form

  B $\rightarrow \alpha$ A, or

  B $\rightarrow \alpha$ A β where $\beta \Rightarrow^* \epsilon$:

  Add FOLLOW(B) to FOLLOW(A)

Example 1

$S \rightarrow a\,S\,e \mid B$

$B \rightarrow b\,B\,C\,f \mid C$

$C \rightarrow c\,C\,g \mid d \mid \varepsilon$

FIRST (S) = { a, b, c, d, ε }

FIRST (B) = { b, c, d, ε }

FIRST (C) = { c, d, ε }

FOLLOW (S) = { EOF, e }

FOLLOW (B) = { EOF, e, f, c, d }

FOLLOW (C) = { f, g, EOF, e, c, d }

Quiz: FOLLOW Sets

$\mathbf{S} \rightarrow \mathbf{a}\ \mathbf{X}\ \mathbf{Y}$

$\mathbf{X} \rightarrow \mathbf{b} \mid \epsilon$

$\mathbf{Y} \rightarrow \mathbf{X}\ \mathbf{c} \mid \mathbf{c}$

Compute the FOLLOW sets of all non-terminals. What is the sum of the sizes of these sets?

Quiz: FOLLOW Sets

$S \rightarrow a\,X\,Y$

$X \rightarrow b \mid \varepsilon$

$Y \rightarrow X\,c \mid c$

FOLLOW(S) = { EOF }

FOLLOW(X) = { c, b }

FOLLOW(Y) = { EOF }

FIRST (S) = { a }

FIRST (X) = { b, ε }

FIRST (Y) = { b, c }

$\sum = 4$

PREDICT Sets

PREDICT set for a rule: Which terminals to look for in LL(1) parser

- If next input token is in PREDICT of rule, apply the rule
- Computing the PREDICT set for rule A $\rightarrow \alpha$:
  - If ϵ in FIRST(α):

    PREDICT(A $\rightarrow \alpha$) = (FIRST(α) - $\{ \epsilon \}$) $\cup$ FOLLOW(A)
  - Otherwise:

    PREDICT(A $\rightarrow \alpha$) = FIRST(α)

Example

Grammar:

S $\rightarrow$ `a` **B**

S $\rightarrow$ `b` **C**

B $\rightarrow$ `b b` **C**

C $\rightarrow$ `c c`

	FIRST	FOLLOW
S	a, b	EOF
B	b	EOF
C	c	EOF

Example

Grammar:	PREDICT:
S → a B	{ a }
S → b C	{ b }
B → b b C	{ b }
C → c c	{ c }

	FIRST	FOLLOW
S	a, b	EOF
B	b	EOF
C	c	EOF

Example

Grammar:	PREDICT:
S → a B	{ a }
S → b C	{ b }
B → b b C	{ b }
C → c c	{ c }

	FIRST	FOLLOW
S	a, b	EOF
B	b	EOF
C	c	EOF

```
S() {
  if (inputToken == a)
    match(a); B();
  else if (inputToken == b)
    match(b); C();
  else error();
}
B() {
  if (inputToken == b)
    match(b); match(b); C();
  else error()
}
C() {
  if (inputToken == c)
    match(c); match(c);
  else error()
}
```

Example

Grammar:	PREDICT:
S → a B	{ a }
S → b C	{ b }
B → b b C	{ b }
C → c c	{ c }

	FIRST	FOLLOW
S	a, b	EOF
B	b	EOF
C	c	EOF

```
S() {
  if (inputToken == a)
    match(a); B();
  else if (inputToken == b)
    match(b); C();
  else error();
}
B() {
  if (inputToken == b)
    match(b); match(b); C();
  else error()
}
C() {
  if (inputToken == c)
    match(c); match(c);
  else error()
}
```

Example

Grammar:	PREDICT:
S → a B	{ a }
S → b C	{ b }
B → b b C	{ b }
C → c c	{ c }

	FIRST	FOLLOW
S	a, b	EOF
B	b	EOF
C	c	EOF

```
S() {
  if (inputToken == a)
    match(a); B();
  else if (inputToken == b)
    match(b); C();
  else error();
}
B() {
  if (inputToken == b)
    match(b); match(b); C();
  else error()
}
C() {
  if (inputToken == c)
    match(c); match(c);
  else error()
}
```

Computing the Parse Table

Computing an LL(1) parse table

- Given: PREDICT set of each rule
- Table is a mapping M:

  $N \times T \rightarrow$ Production rule or error

- For all productions A $\rightarrow \alpha$ do
  - For each terminal t in PREDICT(A $\rightarrow \alpha$):

    M[A][t] = A $\rightarrow \alpha$

  - Every undefined table entry is an error

Example: Parse Table

Non-term. \ Term.	a	b	c
S	1	2	–
B	–	3	–
C	–	–	4

Table-based, Predictive Parsing

```
stack.push(EOF); stack.push(startSymbol);
nextToken = lookAhead();
repeat
  x = stack.pop();
  if x is terminal or EOF
    if x == nextToken
      nextToken = lookAhead()
    else error()
  else // x is non-terminal
    if M[x][nextToken] == x -> y1 y2 .. ym
      stack.push(ym); ...; stack.push(y1);
    else error()
until x is EOF
```

Table-based, Predictive Parsing

Parse stack: Prediction of what will be seen in the future

```
stack.push(EOF); stack.push(startSymbol);
nextToken = lookAhead();
repeat
  x = stack.pop();
  if x is terminal or EOF
    if x == nextToken
      nextToken = lookAhead()
    else error()
  else // x is non-terminal
    if M[x][nextToken] == x -> y1 y2 .. ym
      stack.push(ym); ...; stack.push(y1);
    else error()
until x is EOF
```

Table-based, Predictive Parsing

```
stack.push(EOF); stack.push(startSymbol);
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repeat
  x = stack.pop();
  if x is terminal or EOF
    if x == nextToken
      nextToken = lookAhead()
    else error()
  else // x is non-terminal
    if M[x][nextToken] == x -> y1 y2 .. ym
      stack.push(ym); ...; stack.push(y1);
    else error()
until x is EOF
```

Read one token after another, always looking only one token ahead

Table-based, Predictive Parsing

Check if expected terminal is indeed the next token

```
stack.push(EOF); stack.push(startSymbol);
nextToken = lookAhead();
repeat
  x = stack.pop();
  if x is terminal or EOF
    if x == nextToken
      nextToken = lookAhead()
    else error()
  else // x is non-terminal
    if M[x][nextToken] == x -> y1 y2 .. ym
      stack.push(ym); ...; stack.push(y1);
    else error()
until x is EOF
```

Table-based, Predictive Parsing

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stack.push(EOF); stack.push(startSymbol);
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  else // x is non-terminal
    if M[x][nextToken] == x -> y1 y2 .. ym
      stack.push(ym); ...; stack.push(y1);
    else error()
until x is EOF
```

Apply a production rule: Push right-hand side onto stack

Table-based, Predictive Parsing

```
stack.push(EOF); stack.push(startSymbol);
nextToken = lookAhead();
repeat
  x = stack.pop();
  if x is terminal or EOF
    if x == nextToken
      nextToken = lookAhead()
    else error()
  else // x is non-terminal
    if M[x][nextToken] == x -> y1 y2 .. ym
      stack.push(ym); ...; stack.push(y1);
    else error()
until x is EOF
```

No entry in table:

Raise error

Example: Table-based Parsing

Input: bcc

Stack	Remaining input	Steps
EOF, S	b, c, c, EOF	Pop S Use rule S → bC Push C, b
EOF, C, b	b, c, c, EOF	Pop b Read next token
EOF, C	c, c, EOF	Pop C Use rule C → cc Push c, c
EOF, c, c	c, c, EOF	Pop c Read next token
EOF, c	c, EOF	Pop c Read next token
EOF	EOF	Pop EOF → Done ✓

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- **Scanning**
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Bottom-up Parsing

- **LR(k) parsers**
  - Left-to-right scanning, Right-most derivation, k tokens look-ahead
- **Difficult to do by hand**
- **Mostly based on automatically generated table**

Shift-reduce Algorithm

- **Repeat until all tokens read and all symbols reduced to start symbol:**
  - Shift (i.e., read) input tokens
  - Try to reduce a group of symbols into a single non-terminal

Example: Shift-Reduce Parsing

$S \rightarrow a\ T\ R\ e$

$T \rightarrow T\ b\ c \mid b$

$R \rightarrow d$

Input: a b b c d e

Steps:

Shift a, shift b

Reduce $T \rightarrow b$

Shift b, shift c

Reduce $T \rightarrow T\ b\ c$

Shift d

Reduce $R \rightarrow d$

Shift e

Reduce $S \rightarrow a\ T\ R\ e$

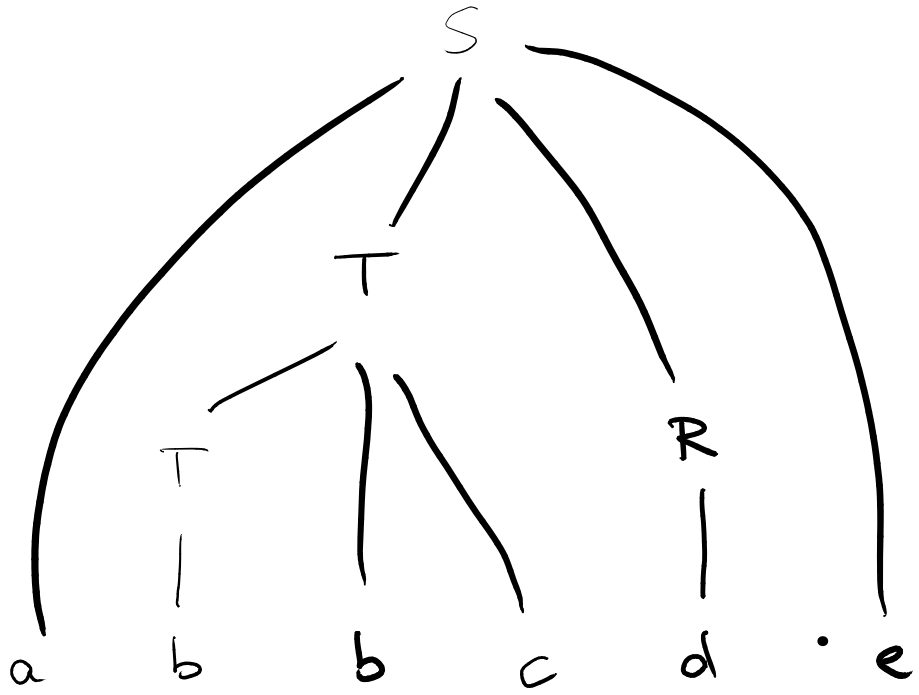

Table-based LR Parsing

- **Two tables**
  - Action table:

    state $\times$ T $\rightarrow$ reduce/shift/accept/error
  - Goto table:

    state $\times$ N $\rightarrow$ state
- **Stack of symbol/state pairs**
  - Record of what has been seen in the past

Example: LR(1) Table

State	a	b	c	d	e	EOF	S	T	R
s0	s1								
s1		s3						2	
s2		s5		s6					4
s3		r3		r3					
s4					s7				
s5			s8						
s6					r4				
s7						acc.			
s8		r2		r2					

Example: LR(1) Table

State	a	b	c	d	e	EOF	S	T	R
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s1		s3						2	
s2		s5		s6					4
s3		r3		r3					
s4					s7				
s5			s8						
s6					r4				
s7						acc.			
s8		r2		r2					

Action table

Example: LR(1) Table

State	a	b	c	d	e	EOF	S	T	R
s0	s1								
s1		s3						2	
s2		s5		s6					4
s3		r3		r3					
s4					s7				
s5			s8						
s6					r4				
s7						acc.			
s8		r2		r2					

Goto table

Example: LR(1) Table

State	a	b	c	d	e	EOF	S	T	R
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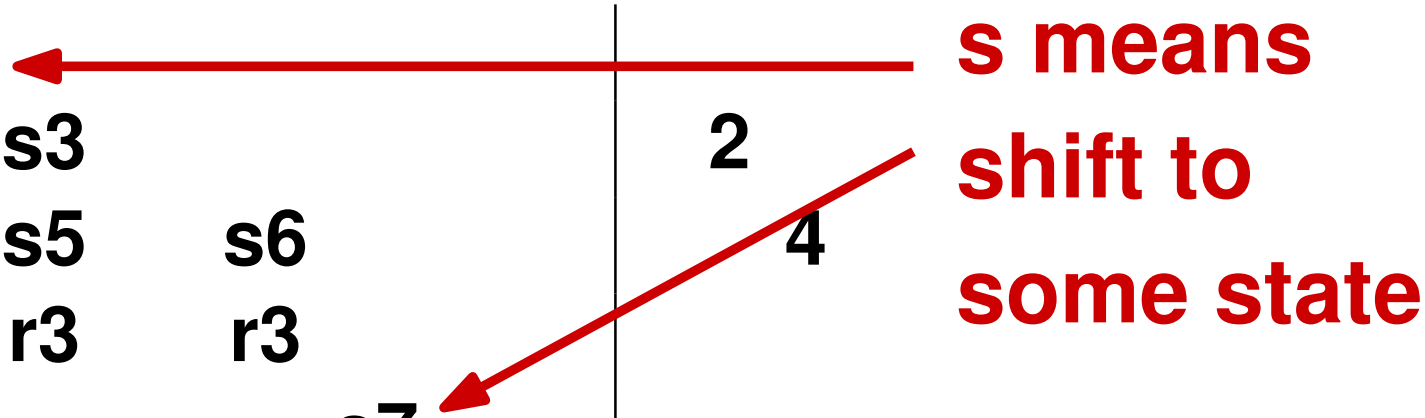
s4					s7				
s5			s8						
s6					r4				
s7						acc.			
s8		r2		r2					



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s3		r3		r3					
s4					s7				
s5			s8						
s6					r4				
s7						acc.			
s8		r2		r2					

r means reduce using some production

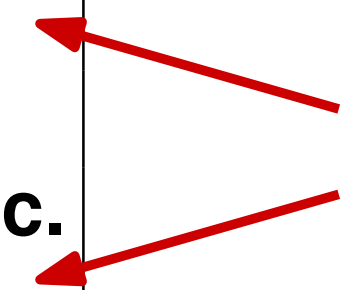
Example: LR(1) Table

State	a	b	c	d	e	EOF	S	T	R
s0	s1								
s1		s3						2	
s2		s5		s6					4
s3		r3		r3					
s4					s7				
s5			s8						
s6					r4				
s7						acc.			
s8		r2		r2					

Accept input (i.e., done with parsing)

Example: LR(1) Table

State	a	b	c	d	e	EOF	S	T	R
s0	s1								
s1		s3						2	
s2		s5		s6					4
s3		r3		r3					
s4					s7				
s5			s8						
s6					r4				
s7						acc.			
s8		r2		r2					



No entry means error

Table-based LR(1) Parsing

```
stack.push(EOF, 0);
nextToken = lookAhead();
repeat
  s = state on top of stack
  if action[s, nextToken] == shift s'
    stack.push(nextToken, s');
    nextToken = lookAhead();
  else if action[s, nextToken] == reduce x -> y1 .. ym
    pop m pairs from stack
    s' = state on top of stack
    stack.push(x, goto[s', x])
  else if action[s, nextToken] == accept
    accept and return
  else error()
```

Table-based LR(1) Parsing

Stack hold roots of partial trees found so far

```
stack.push(EOF, 0);
nextToken = lookAhead();
repeat
   s = state on top of stack
   if action[s, nextToken] == shift s'
      stack.push(nextToken, s');
      nextToken = lookAhead();
   else if action[s, nextToken] == reduce x -> y1 .. ym
      pop m pairs from stack
      s' = state on top of stack
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Table-based LR(1) Parsing

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  else if action[s, nextToken] == reduce x -> y1 .. ym
    pop m pairs from stack
    s' = state on top of stack
    stack.push(x, goto[s', x])
  else if action[s, nextToken] == accept
    accept and return
  else error()
```

Reduce partial trees into a non-terminal by applying a rule

Table-based LR(1) Parsing

```
stack.push(EOF, 0);
nextToken = lookAhead();
repeat
  s = state on top of stack
  if action[s, nextToken] == shift s'
    stack.push(nextToken, s');
    nextToken = lookAhead();
  else if action[s, nextToken] == reduce x -> y1 .. ym
    pop m pairs from stack
    s' = state on top of stack
    stack.push(x, goto[s', x])
  else if action[s, nextToken] == accept
    accept and return
  else error()
```

Read another token

Table-based LR(1) Parsing

```
stack.push(EOF, 0);
nextToken = lookAhead();
repeat
  s = state on top of stack
  if action[s, nextToken] == shift s'
    stack.push(nextToken, s');
    nextToken = lookAhead();
  else if action[s, nextToken] == reduce x -> y1 .. ym
    pop m pairs from stack
    s' = state on top of stack
    stack.push(x, goto[s', x])
  else if action[s, nextToken] == accept
    accept and return
  else error()
```

All subtrees reduced to start symbol

How to Get the Table?

- **Using a "characteristic finite-state machine" computed from the grammar**
- **Details differ for different kinds of LR parsers**
  - SLR (simple LR)
  - LALR (look-ahead LR)
  - Full-LR
- **Beyond the scope of this course**

Quiz: Parsing

Which of these statements is true?

- LR(k) parsers build a parse tree from the top down.
- A table-driven LL parser uses PREDICT sets to decide which rule to apply.
- The PREDICT set is the intersection of the FIRST and FOLLOW sets.
- The “shift” operation reads another input token.

Quiz: Parsing

Which of these statements is true?

- ~~LR(k) parsers build a parse tree from the top down~~.
- A table-driven LL parser uses PREDICT sets to decide which rule to apply.
- ~~The PREDICT set is the intersection of the FIRST and FOLLOW sets~~.
- The “shift” operation reads another input token.

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