Programming Paradigms

Functional Languages

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Overview

- Introduction
- A Bit of Scheme
- Evaluation Order

Wake-up Quiz

What does the following Scheme code evaluate to?

```
(let ((y 2))
  (let ((y 4)
        (x y))
    (- y x)))
```

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`let` **binds names to values**

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`let` binds names to values

Scope of bindings: Second argument only

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  (let ((y 4)
        (x y))
    (- y x)))
```

`let` binds names to values

`x` takes the value of the outer `y`

Scope of bindings: Second argument only

Result: 2

Functional Languages

- **Functional paradigm: Alternative to imperative PLs**
  - Output: Mathematical function of input
  - No internal state, no side effects
- **In practice: Fuzzy boundaries**
  - “Functional” features in many “imperative” PLs
    - E.g., higher-order functions
  - “Imperative features” in many “functional” PLs
    - E.g., assignment and iteration

Historical Origins

- **Lambda calculus**
    - Alonzo Church, 1930s
- **Express computation based on**
    - Abstraction into functions
        - E.g., $(\lambda x.M)$
    - Function application
        - E.g., $(M\ N)$

Features

- First-class function values and higher-order function
- Extensive polymorphism
- List types and operators
- Structured function returns
- Constructors for structured objects
- Garbage collection

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Functions assigned to variables, passed as arguments, or return values

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Use a function on different kinds of values, e.g., using type inference

Features

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- Extensive polymorphism
- List types and operators
- Structured function returns
- Constructors for structured objects
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Ideal for recursion (handle first element and then recursively the remainder)

Features

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Functions can return any structured data, e.g., lists and functions

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- Extensive polymorphism
- List types and operators
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- Constructors for structured objects
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Construct aggregate objects inline and all-at-once

Features

- First-class function values and higher-order function
- Extensive polymorphism
- List types and operators
- Structured function returns
- Constructors for structured objects
- Garbage collection

Necessary because evaluation tends to create lots of temporary data

Purely Functional PLs

- **Functions depend only on their parameters**
  - Not on any other global or local state
  - Order of evaluation is irrelevant
    - Eager and lazy evaluation yield same result
- **E.g., Haskell**
  - By Philip Wadler et al., first released in 1990
  - Actively used as a research language

Non-Pure Functional PLs

- **Mix of functional features with assignments**
- **E.g., Scheme**
  - Dialect of Lisp
  - By Guy Steele and Gerlad Jay Sussman (MIT)
- **E.g., OCaml**
  - Extends ML with OO features
  - Developed at INRIA (France)

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Function Application

- **Pair of parentheses: Function application**
  - First expression inside: Function
  - Remaining expressions: Arguments
- **Examples:**

```
(+ 3 4)                          ((+ 3 4))
```

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Applies + function
to 3 and 4.
Evaluates to 7.

Function Application

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- **Examples:**

```
(+ 3 4)
```

Applies + function to 3 and 4.
Evaluates to 7.

```
((+ 3 4))
```

Tries to call 7 with zero arguments.
Gives runtime error.

Creating Functions

- **Evaluating a lambda expression yields a function**
  - First argument to `lambda`: Formal parameters
  - Remaining arguments: Body of the function
- **Example:**

```
(lambda (x) (* x x))
```

Creating Functions

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  - First argument to `lambda`: Formal parameters
  - Remaining arguments: Body of the function
- **Example:**

```
(lambda (x) (* x x))
```

Yields the “square” function

Bindings

- **Names bound to values with `let`**
  - ☐ First argument: List of name-value pairs
  - ☐ Second argument: Expressions to be evaluated in order
- **Example:**

```
(let ((a 3)
      (b 4)
      (square (lambda (x) (* x x)))
      (plus +))
   (sqrt (plus (square a) (square b))))
```

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```

Yields 5.0

Conditional Expressions

- **Simple conditional expression with `if`**
  - ☐ First argument: Condition
  - ☐ Second/third argument: Value returned if condition is true/false
- **Multiway conditional expression with `cond`**
- **Examples:**

```
(if (< 2 3) 4 5)
```

```
(cond
  ((< 3 2) 1)
  ((< 4 3) 2)
  (else 3))
```

Conditional Expressions

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```

Yields 4

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- **Multiway conditional expression with `cond`**
- **Examples:**

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(if (< 2 3) 4 5)
```

Yields 4

```
(cond
  ((< 3 2) 1)
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  (else 3))
```

Yields 3

Dynamic Typing

- Types are determined and checked at runtime
- Examples:

```
(if (> a 0) (+ 2 3) (+ 2 "foo"))
```

```
(define min (lambda (a b) (if (< a b ) a b)))
```

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Evaluates to 5 if `a` is positive; runtime type error otherwise

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Evaluates to 5 if `a` is positive;
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Implicitly polymorphic:
Works both for integers and floats.

Quiz: Scheme Examples

Which of the following yields 7?

```
; Program 1
2 + 5

; Program 2
((lambda (a b) (- b a)) 2 9)

; Program 3
((* 1 (+ 4 3)))

; Program 4
(if (> 1 2) (+ 3 4) 5)
```

Quiz: Scheme Examples

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2 + 5   ✗

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((lambda (a b) (- b a)) 2 9)   ✔

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((* 1 (+ 4 3)))   ✗

; Program 4
(if (> 1 2) (+ 3 4) 5)   ✗
```

Lists

- **Central data structure with various operations**
    - `car` extracts first element
    - `cdr` extracts all elements but first
    - `cons` joins a head to the rest of a list
- **Examples:**

```
(car '(2 3 4))     (cdr '(2 3 4))     (cons 2 '(3 4))
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”Quote” to prevent interpreter from evaluating (i.e., a literal)

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Yields (3 4)

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Yields 2 **Yields (3 4)** **Yields (2 3 4)**

Assignments

- **Side effects via**
  - ☐ `set!` for assignment to variables
  - ☐ `set-car!` for assigning head of list
  - ☐ `set-cdr!` for assigning tail of list
- **Example:**

```
(let ((x 2)
      (l '(a b)))
  (set! x 3)
  (set-car! l '(c d))
  (set-cdr! l '(e))
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```

Yields (3 (c d) e)

$x = 2$

$l = (a \ b)$

head of list

$x = 3$

$l = ((c \ d) \ b)$

tail of list

$l = ((c \ d) \ e)$

$\rightarrow (3 \ (c \ d) \ e)$

Sequencing

- Cause interpreter to evaluate multiple expressions one after another with `begin`
- Example:

```
(let
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  (begin
    (display "hi ")
    (display n)))
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Prints ”hi there”

Iteration

- Several forms of loops, e.g., with `do`
- Example:

```
((lambda (n)
  (do ((i 0 (+ i 1))
       (a 0 b)
       (b 1 (+ a b)))
    ((= i n) b)
    (display b)
    (display " "))) 5)
```

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List of triples that each

- specify a new variable
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Body of the loop

Computes first n Fibonacci numbers

Programs as Lists

- **Programs and lists: Same syntax**
    - Both are S-expressions: String of symbols with balanced parentheses
- **Construct and manipulate an unevaluated program as a list**
- **Evaluate with `eval`**
- **Example:**

```
(eval (cons '+ (list '2 '3)))
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Constructs a list from the given arguments

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Yields 5

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Evaluation Order

- **In what order to evaluate subcomponents of an expression?**
  - Applicative-order: Evaluate arguments before passing them to the function
  - Normal-order: Pass arguments unevaluated and evaluate once used
- **Scheme uses applicative-order**

```
(define double (lambda (x) (+ x x)))
```

Applicative-order

```
(double (* 3 4))
=> (double 12)
=> (+ 12 12)
=> 24
```

Normal order

```
(double (* 3 4))
=> (+ (* 3 4) (* 3 4))
=> (+ 12 (* 3 4))
=> (+ 12 12)
=> 24
```

Doing extra work with normal order!

```
(define switch (lambda (x a b c)
    (cond ((< x 0) a)
          ((= x 0) b)
          ((> x 0) c))))
```

→ Doing extra work with applicative order!

Applicative-order

```
(switch -1 (+ 1 2) (+ 2 3) (+ 3 4))
⇒ (switch -1 3 (+ 2 3) (+ 3 4))
⇒ (switch -1 3 5 (+ 3 4))
⇒ (switch -1 3 5 7)
⇒ (cond ((< -1 0) 3)
        ((= -1 0) 5)
        ((> -1 0) 7))
⇒ ... ⇒ 3
```

Normal-order

```
(switch -1 (+ 1 2) (+ 2 3) (+ 3 4))
⇒ (cond ((< -1 0) (+ 1 2))
        ((= -1 0) (+ 2 3))
        ((> -1 0) (+ 3 4)))
⇒ (cond (#t (+ 1 2))
          ⋮
⇒ 3
```

Impact on Correctness

- **Evaluation order also affects correctness**
- **E.g., runtime error when evaluating an ”unneeded” subexpression**
  - Terminates program in applicative-order
  - Not noticed in normal-order

Lazy Evaluation

- **Evaluate subexpressions on-demand**
- **Avoid re-evaluating the same expression**
  - Memorize its result
- **Transparent to programmer only in PL without side effects, e.g., Haskell**
  - In PLs with side effects, e.g., Scheme: Programmer can explicitly ask for lazy evaluation with `delay`

Quiz: Evaluation Order

```
(define diff (lambda (x y) (- x y)))
(define f (lambda (x) (* x (+ 1 x))))
```

How many evaluation steps are needed to evaluate

`(f (diff 3 4))`

under applicative-order and normal-order evaluation?

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5 and 7

Applicative order

(f (diff 3 4))
⇒ (f (- 3 4))
⇒ (f -1)
⇒ (* -1 (+ 1 -1))
⇒ (* -1 0)
⇒ 0

5 steps

Normal order

(f (diff 3 4))
⇒ (* (diff 3 4) (+ 1 (diff 3 4)))
⇒ (* (- 3 4) (+ 1 (diff 3 4)))
⇒ (* -1 (+ 1 (diff 3 4)))
⇒ (* -1 (+ 1 (- 3 4)))
⇒ (* -1 (+ 1 -1))
⇒ (* -1 0)
⇒ 0

7 steps

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