Type Safety with JSON Subschema

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Abstract

JSON is a popular data format used pervasively in web APIs, cloud computing, NoSQL databases, and increasingly also machine learning. JSON Schema is a language for declaring the structure of valid JSON data. There are validators that can decide whether a JSON document is valid with respect to a schema. Unfortunately, like all instance-based testing, these validators can only show the presence and never the absence of a bug. This paper presents a complementary technique: JSON subschema checking, which can be used for static type checking with JSON Schema. Deciding whether one schema is a subschema of another is non-trivial because of the richness of the JSON Schema specification language. Given a pair of schemas, our approach first canonicalizes and simplifies both schemas, then decides the subschema question on the canonical forms, dispatching simpler subschema queries to type-specific checkers. We apply an implementation of our subschema checking algorithm to 8,548 pairs of real-world JSON schemas from different domains, demonstrating that it can decide the subschema question for most schema pairs and is always correct for schema pairs that it can decide. We hope that our work will bring more static guarantees to hard-to-debug domains, such as cloud computing and artificial intelligence.

1 Introduction

JSON (JavaScript Object Notation) is a data serialization format that is widely adopted to store data on disk or send it over the network. Derived from JavaScript, JSON is both human- and machine-readable, and there are now JSON parsers for many programming languages. JSON supports primitive data types, such as strings, numbers, and Booleans, and two data structures: arrays, which represent lists of values, and objects, which represent maps of key-value pairs. The data types can be nested, e.g., to have an array of two objects that each map a key to some primitive value.

JSON is used in numerous applications. It is the most popular data exchange format in web APIs, ahead of XML [21]. Cloud-hosted applications also use JSON pervasively, e.g., in micro-services that communicate via JSON data [17]. On the data storage side, not only do traditional database management systems, such as Oracle, IBM DB2, MySQL, and PostgreSQL, now support JSON, but two of the most widely deployed NoSQL database management systems, MongoDB and Cloudant, are entirely based on JSON. Beyond web, cloud, and database applications, JSON is also gaining adoption in artificial intelligence (AI) [13, 23].

With the wide adoption of JSON as a data serialization format soon emerged the need for a way to describe how a JSON document should look. For example, a web API that consumes JSON data can avoid unexpected behavior if it knows the structure of the data it receives. To describe a JSON document, *JSON Schema* allows users to declaratively define the structure of nested values (documents) via types (schemas) [19]. We adopt the value space definition for *type* as being a set of possible values [18]. A JSON Schema *validator* checks whether a JSON document d conforms to a schema s, denoted $d : s$. There are libraries with JSON Schema validators for many programming languages, and they are widely used to make software more reliable.

Since these $d : s$ validation checks happen at runtime, they can usually only detect problems late during deployment and production. In cloud applications, a wrongly-structured document can cause hard-to-debug failures, it may go unnoticed and simply cause undesired behavior, or even worse, it may exploit a security vulnerability. In AI, a machine learning *pipeline* is a graph of operators for preprocessing and prediction [8]. Any mismatch between training data and a pipeline, between production data and training data, or between data in adjacent steps of a pipeline can cause crashes or poor predictive performance [7]. Sometimes, dynamic data validity checks of the form $d : s$ only trigger after earlier pipeline steps have already performed costly computation.

Given how essential dynamic $d : s$ checks are, we argue that JSON schemas could be even more useful if they were also checked statically. The quintessential static property is whether one schema is a *subschema* of another. We say that a schema s is a subschema of a schema t, denoted $s <: t$, if all documents that validate against s also validate against t. Subschema checks of the form $s <: t$ can find the same mistakes as dynamic schema validation, but with less wasted compute and human time, as they can statically rule out entire classes of errors.

JSON schemas have several features that make subschema checking difficult. Schemas for primitive types involve non-trivial features, such as regular expressions for strings and `multipleOf` for numbers. Schemas for compound types involve `uniqueItems` constraints for arrays and regular expressions for object property names. Furthermore, JSON schemas can be composed using schema conjunction, disjunction, and negation, which requires handling of complex, compound types. Finally, the `enum` type is entangled with other types, further exacerbating the above-mentioned difficulties.

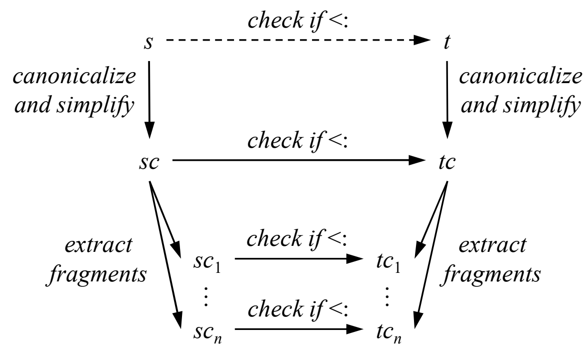

Figure 1. Overview of JSON subschema checker.

This paper presents *jsonsubschema*, the first subschema checker for JSON Schema that handles all the above-listed difficult features (and thus, almost all features used in real-world JSON schemas, c.f. Section 6.4). Figure 1 gives a high-level overview of our algorithm. To check whether $s <: t$, the checker first canonicalizes and simplifies s to sc and t to tc. This preprocessing disentangles different cases to consider during subschema checking, while preserving the schema semantics. Next, our algorithm extracts corresponding fragments sc_i and tc_i. These fragments are type-homogeneous, i.e., they refer to only one basic JSON type each. This homogeneity makes it possible to use separate modules to check whether $sc_i <: tc_i$ for each type. The question whether JSON subschema checking is decidable did not have an obvious answer before our work. This paper serves as a constructive argument that yes, JSON subschema checking is decidable.

The most closely related prior work is an open-source project called is-json-schema-subset developed concurrently with our work, but it only handles a small fraction of the features of JSON Schema [12]. The problem of subschema checking has also been explored for XML, where it is called schema containment [24]. However, that approach treats XML as tree automata, which has been shown to be impossible for JSON [19].

In summary, this paper makes the following contributions:

- A canonicalizer and simplifier that converts a schema s into a schema sc that is simpler to check yet permits the same set of documents (Sections 4.1 and 4.2).
- A subschema checker for canonicalized JSON schemas that uses separate subschema checking modules for each basic JSON type (Section 4.3).
- Empirical evidence on 8,548 pairs of JSON schemas taken from real-world applications in the web, cloud computing, and AI. We show that the implementation of our algorithm successfully answers the subschema question in most cases, and when it does, yields correct answers in reasonable time (Section 6).

The code for our JSON subschema checker is included in the supplemental material for this paper submission. We also have plans for an artifact submission and an open-source release. While there are some esoteric cases that our implementation does not yet handle, our evaluation demonstrates that our subschema checker handles virtually all common cases occurring in practice, and this paper includes a discussion of how to tackle the remaining cases. Overall, we hope that our work helps bring the blessings of static type safety to applications based on JSON and JSON Schema, e.g., in cloud computing and AI systems.

2 Background

This section briefly describes JSON data and JSON Schema validation.

2.1 JSON Data

JSON was conceived as a light-weight, text-based, and programming language-agnostic data interchange format [15].

Definition 2.1 (*Jprimitive*)**.** The primitive JSON types are:

$$Jprimitive = \{\texttt{integer}, \texttt{number}, \texttt{string}, \texttt{boolean}, \texttt{null}\}$$

Besides primitive types, JSON has two structured types: ordered lists of values (arrays) and unordered maps of key-value pairs (objects).

Definition 2.2 (*Jstructure*)**.** The structured JSON types are:

$$Jstructure = \{\texttt{array}, \texttt{object}\}$$

Definition 2.3 (*Jtypes*)**.** The set of JSON data types is:

$$Jtypes = Jprimitive \cup Jstructure$$

A *valid JSON document* (or value) is either a value of one of the basic types, or an array whose elements are valid JSON documents, or an object mapping string keys to valid JSON documents. Some examples of valid JSON documents include `5`, `null`, `'ab'`, `[]`, `[.5, {}, 'a']`, and `{'foo':1, 'bar':[true, '']}`.

Finally, JSON documents can use JSON references to point to data from the same or other JSON documents.

2.2 JSON Schema

JSON Schema is a declarative language for defining the structure and permitted values of a JSON document [26]. JSON Schema itself uses JSON syntax. JSON Schema is an Internet Draft from the Internet Engineering Task Force (IETF). It is continuously evolving, currently at draft-2019-09. However, in this work we focus on draft04 [11], one of the most widely adopted versions of JSON Schema.

To specify which data types are allowed, JSON Schema uses the keyword `'type'` with one type name or a list of type names. For example, schema `{'type':'string'}` accepts strings and schema `{'type':['null','boolean']}` accepts `null` or Boolean values. Each JSON type has a set of validation *keywords* that restrict the values a schema of this type permits. For instance, schemas for `integer` or `number` values can use the keywords `minimum` and `maximum` to restrict the range of allowed values, while `string` schemas can use the

Table 1. Keywords of JSON Schema.

JSON type	**Type-specific keywords**
`string`	`minLength`, `maxLength`, `pattern`
`integer`, `number`	`minimum`, `exclusiveMinimum`, `maximum`, `exclusiveMaximum`, `multipleOf`
`boolean`	–
`null`	–
`array`	`items`, `additionalItems`, `minItems`, `maxItems`, `uniqueItems`
`object`	`properties`, `patternProperties`, `additionalProperties`, `minProperties`, `maxProperties`, `required`, `dependencies`
Any-schema keywords	`type`, `enum`, `$ref`
Boolean connectives	`anyOf`, `allOf`, `oneOf`, `not`

keyword `pattern` to define regular expressions of allowed strings. The upper part of Table 1 lists the set of keywords associated with each JSON type.

In addition to type-specific keywords, JSON Schema allows enumerating exact values with `enum` and combining different schemas using a set of Boolean connectives (Table 1, lower part). For example, schema `{'enum':['a', [], 1]}` restricts the set of permitted JSON values to the string literal `'a'`, an empty array, or the integer 1. Boolean connectives, such as `anyOf`, `allOf`, and `not`, allow schema writers to express disjunctions, conjunctions, and negations of schemas. Finally, the keyword `$ref` retrieves schemas using URIs and JSON pointers. JSON validation against a schema with `$ref` has to satisfy the schema retrieved from the specified URI or JSON pointer. We refer the interested reader to the full specification of JSON Schema [11] and its formalization [19].

Given a JSON document d and a JSON schema s, schema validation checks whether d conforms to s.

Definition 2.4 (JValid(d, s))**.** For any JSON document d and any JSON schema s, $\text{JValid}(d, s) \longrightarrow \{True, False\}$, also written $d : s$, decides whether d is valid with respect to s.

This decision problem JValid(d, s) is shown to be PTIME-hard [19] and solvable in linear time when eliminating the `uniqueItems` keyword as it involves sorting. JSON validators have been implemented in most major programming languages and are widely used in several domains.

3 Problem Statement

Given two schemas s and t, our approach decides whether s is a subschema of t. This section defines the subschema relation, gives concrete usage scenarios for it, and describes why deciding the subschema question is non-trivial.

3.1 JSON Subschema

A JSON schema defines the *type* of valid JSON documents and validation corresponds to a runtime check whether a JSON document d is of type s. But what if we want to reason about JSON schemas statically? Assume we have two JSON schemas s and t, and a JSON document d such that $d : s$. The decision procedure JValid(d, t) can answer the question whether $d : t$ for a dynamically given d but not statically. Moreover, generalizing our knowledge about schemas s and t beyond a specific JSON document d is possible only by enumerating all documents that conform to s and t. The goal of this work is to tell whether the set of documents that conform to schema s is a subset of the set of documents that conform to schema t. We aim at answering this question statically, i.e., without enumerating all valid documents, and we call this question the *JSON Subschema* decision problem.

Definition 3.1 (JSubSchema <:)**.** For any two JSON schemas s and t, the subschema relation, denoted <:, is defined as:

$$s <: t \iff \big(\forall d : \text{JValid}(d, s) \implies \text{JValid}(d, t)\big)$$

The relation <: is a partial order that is reflexive, transitive, and anti-symmetric. Equivalence of schemas follows directly by the anti-symmetry of the subtype relation.

Definition 3.2 (JEquivSchema ≡)**.** For any two JSON schemas s and t, the equivalence relation $s \equiv t$ is given by:

$$s \equiv t \iff (s <: t \land t <: s)$$

That is, we say that two schemas are equivalent if they describe the same exact set of JSON documents.

3.2 Usage Scenarios

Backward compatibility. One of the most pervasive use cases of JSON schemas is describing requests and responses of web APIs. For example, version 0.6.1 of the Washington Post `ans-schema` contains the following:

```
'category':
  {'type': 'string',
   'enum': ['staff', 'wires', 'freelance', 'other' ]}
```

The continuous development and evolution of these APIs involves regular changes to the corresponding JSON schemas, and developers need to keep a close eye on such changes to avoid breaking backward compatibility. For example, version 0.6.2 of the same schema contains:

```
'category': {
  'type': 'string',
  'enum': ['staff', 'wires', 'freelance',
           'stock', 'handout', 'other' ]}
```

Version 0.6.1 is a subschema of version 0.6.2. Assuming the developers intend to retain backward compatibility, this evolution would be fine for an API request argument, but it could break clients when used as an API response.

```
trainable = sklearn.pipeline.make_pipeline(
    RFE(estimator=Forest(n_estimators=100)), NMF())
```

```
%%time
try:
    trainable.fit(train_X, train_y)
except ValueError as e:
    message = str(e)
print(message, file=sys.stderr)
```

```
CPU times: user 42.5 s, sys: 969 ms, total: 43.4 s
Wall time: 43.9 s
```

```
Negative values in data passed to NMF (input X)
```

Figure 2. AI pipeline with slow dataset format exception.

Schemas as static types. JSON subschema checking can help make code more robust by flagging some mistakes statically. Consider version 0.14.0 of the `NodeAddress` schema from Kubernetes, extracted from its OpenAPI specification:

```
{'type': 'object', 'required': ['type', 'address'],
 'properties': {
   'address': {'description': 'The node address.',
               'type': ['string', 'null']},
   'type': {'description': 'Node address type, one of
             Hostname, ExternalIP or InternalIP.',
            'type': ['string', 'null']}}}
```

With this schema, an application can check `NodeAddress` objects at runtime, but runtime errors in distributed, cloud-based systems are difficult to debug. Therefore, client code might define a stricter schema s:

```
{'anyOf': [
 {'type': 'object', 'required': ['type', 'address'],
  'properties': {
    'type': {'enum': ['ExternalIP', 'InternalIP']},
    'address': {'type': 'string',
                'pattern': '^\d+\.\d+\.\d+\.\d+$'}}},
 {'type': 'object', 'required': ['type', 'address'],
  'properties': {
    'type': {'enum': ['Hostname']},
    'address': {'type': 'string',
                'pattern': '^[A-Za-z0-9.]+$'}}}]}
```

Schema s uses `enum`s to constrain the values for `'type'` and `pattern`s to constrain the values for `'address'`, with an `anyOf` to provide two cases. By being stricter, schema s can rule out more bugs. The static check s <: `NodeAddress` can validate that s is indeed a subschema.

Machine learning pipelines. Machine learning pipelines are of little use if the data is formatted incorrectly [7]. For illustration, consider the pipeline in Figure 2, which is a screenshot from a notebook that uses scikit-learn [8]. The pipeline consists of two operators, RFE and NMF. RFE (recursive feature elimination) keeps only those features of the input data that are the most useful according to its argument, a random forest classifier. NMF (non-negative matrix factorization) transforms the data further, e.g., for dimensionality reduction. The call to `fit` causes scikit-learn to first train RFE on the training data, which is computationally expensive. However, if the input data had negative features and RFE did not eliminate them, then the transformed data coming out of RFE still has negative features. Therefore, when scikit-learn subsequently attempts to train NMF, it throws an exception. Since the error only manifests after the costly training of RFE, this example took 43.9 seconds to run.

```
trainable = lale.operators.make_pipeline(
    RFE(estimator=Forest(n_estimators=100)), NMF())
```

```
%%time
try:
    trainable.validate_schema(schema_X, schema_y)
except lale.helpers.SubschemaError as e:
    message = str(e)
print(message, file=sys.stderr)
```

```
CPU times: user 62.5 ms, sys: 0 ns, total: 62.5 ms
Wall time: 71.7 ms
```

```
Expected to_schema(data) to be a subschema of NMF.input_schema_fit().
to_schema(data) = {
    'type': 'object',
    'required': ['X', 'y'],
    'properties': {
        'X': {
            'type': 'array',
            'items': {
                'type': 'array',
                'items': {'type': 'number'}}},
        'y': {
            'type': 'array',
            'items': {'type': 'number'}}}}
NMF.input_schema_fit() = {
    'type': 'object',
    'required': ['X'],
    'properties': {
        'X': {
            'type': 'array',
            'items': {
                'type': 'array',
                'items': {'type': 'number', 'minimum': 0.0}}},
        'y': {}}}
```

Figure 3. AI pipeline with fast dataset schema exception.

Figure 3 shows how subschema checking can speed up the error detection by 600× compared to Figure 2. In the error message, `to_schema(data)` is the schema of the actual data that is piped from RFE to NMF, where `'X'` is an array of arrays of numbers without range constraints. In contrast, `NMF.input_schema_fit` indicates the type expected by NMF, where `'X'` is an array of arrays of numbers with minimum 0. Due to the minimum constraint, the subschema check fails, and the developer detects the bug earlier, before running the actual training. As training a machine learning pipeline may take minutes or even hours, depending on the data size, and as practitioners often try hundreds of pipeline variants [10], static checking can yield a significant productivity boost.

3.3 Challenges

JSON schemas define the nested *structure* and *valid values* permitted in a set of JSON documents. The rich feature set of JSON Schema makes establishing or refuting a subschema relationship between two schemas non-trivial. Even for simple, structurally similar schemas, such as `{'enum':[1, 2]}` and `{'enum':[2, 1]}`, equivalence does not hold through textual equality. There are several challenges for algorithmically checking the subtype relation of JSON schemas.

First, the schema language is flexible and the same set of JSON values, i.e., the same type, could be described in several different syntactical forms, i.e., schemas. For example,

```
{'type': ['string',
          'null'],
 'not': {'enum': ['']}}
```

(a)

```
{'anyOf': [{'type': 'string'},
           {'type': 'null'}],
 'not': {'type': 'string', 'enum': ['']}}
```

(b)

```
{'allOf': [{'anyOf': [{'type': 'string'},
                      {'type': 'null'}]},
           {'not': {'type': 'string', 'enum': ['']}}]}
```

(c)

```
{'allOf': [{'anyOf': [{'type': 'string'}, {'type': 'null'}]},
           {'anyOf': [{'type': 'array'}, {'type': 'object'},
             {'type': 'number'}, {'type': 'integer'}, {'type': 'boolean'},
             {'type': 'null'}, {'type': 'string', 'pattern': '.+'}]}]}
```

(d)

```
{'anyOf':
  [{'type': 'string', 'pattern': '.+'},
   {'type': 'null'}]}
```

(e)

Figure 4. Five syntactically different but semantically equivalent schemas for a value that is either a non-empty string or null.

Figure 4 shows five equivalent schemas describing a JSON value that is either a non-empty string or null.

Second, even for primitive types, such as strings and numbers, nominal subtyping is not applicable. JSON Schema lets users specify various constraints on primitive types, resulting in non-trivial interactions that are not captured by nominal types. For example, one cannot infer that an `integer` schema is a subtype of a `number` schema without properly comparing the range and multiplicity constraints of the schemas.

Third, Boolean connectives combine non-homogeneous types such as `'string'` and `'null'` in Figure 4b. Moreover, enumerations restrict types to predefined values, which require careful handling, especially when enumerations interact with non-enumerative types, such as in Figures 4a, 4b, and 4c.

Fourth, the schema language allows implicit conjunctions and disjunctions. For example, Figure 4b has an implicit top-level conjunction between the subschemas under `anyOf` and `not`. As another example, a schema that lacks a type keyword, such as `{'pattern':'.+'}`, has an implicit disjunction between all possible types, while still enforcing any type-specific keyword, such as the pattern for strings only.

Finally, JSON Schema also allows uninhabited types. That is, a schema can be syntactically valid yet semantically self-contradicting, e.g., `{'type':'number','minimum':5,'maximum':0}`. Such schemas validate no JSON value at all and complicate reasoning about subtyping.

4 Algorithm

This section describes how we address the problem of checking whether one JSON schema is a subtype of another. Because JSON schemas are complex, creating a subtype checker for arbitrary schemas directly would necessitate a complex algorithm to handle all of its variability. Instead, we decompose the problem into three steps. The first step canonicalizes a given schema into an equivalent but more standardized schema (Section 4.1). The second step further simplifies a schema (Section 4.2) by reducing intersection, union, negation, and enumeration of types into either a single type or a disjunction of types. Finally, the third step checks for two canonicalized and simplified schemas whether one is a subtype of the other by extracting and comparing type-homogeneous schema fragments (Section 4.3).

4.1 JSON Schema Canonicalization

To enable the checker to work for all JSON schemas, this section introduces a canonicalization procedure that compiles any JSON schema into an equivalent canonical schema. Formally, given any JSON schema as input, canonicalization terminates and produces a semantically equivalent, canonical JSON schema as output.

The canonicalization enforces three main simplifications. First, JSON Schema supports four Boolean connective keywords *JConnectives* = {`anyOf`, `allOf`, `oneOf`, `not`}, denoting logical disjunction, conjunction, exclusive or, and negation. These connectives can be arbitrarily nested, making a syntax-driven approach to subtype checking challenging. To obviate this challenge, canonicalization converts the schema into an equivalent schema in disjunctive normal form, with negations pushed down through the other connectives.

Second, JSON Schema allows many alternative ways to represent the same thing. Additionally, there are many fields that can be omitted and the defaults are then assumed. Canonicalization picks, when possible, one form, and supplies omitted defaults explicitly.

Third, JSON Schema allows schemas to mix specifications for different types. To enable local domain-specific reasoning in the subtype checker, canonicalization splits up these schemas into smaller, homogeneously typed schemas that are combined using Boolean connectives as appropriate.

As examples of the latter problems, reconsider the schemas in Figure 4. Schema 4a specifies the union using a list of types for the `type` keyword, while the other schemas use `anyOf` to achieve the same effect. Moreover, schemas in Figure 4 enforce non-empty strings using syntactically different constraints: Schemas 4a to 4c negate the `enum` of an empty string, whereas schemas 4d to 4e use a regular expression `pattern`.

We describe our JSON Schema canonicalization as a term rewriting system that operates on valid JSON schemas. To simplify the presentation, we define the following useful sets and functions.

Let *Jkeywords* be the set of all type-specific JSON keywords listed in Table 1. For any JSON schema type $\tau \in$

$$\text{Non-keywords}\ \frac{s.\texttt{type} = \tau \qquad \tau \in \mathit{Jtypes}}{s \longrightarrow \{k \mapsto v \mid k \mapsto v \in s \wedge k \in kw(\tau)\}} \qquad \text{Missing type}\ \frac{\texttt{type} \notin dom(s) \qquad \mathit{JConnectives} \cap dom(s) = \emptyset}{s \longrightarrow \{s.\texttt{type} \mapsto \mathit{Jtypes}\}}$$

$$\text{Homogeneous enum}\ \frac{s.\texttt{enum} = [v_{i_1}, \ldots, v_{i_m}, \ldots, v_{j_1}, \ldots, v_{j_n}] \qquad \mathit{typeOf}(v_i) = \tau_i \qquad \ldots \qquad \mathit{typeOf}(v_j) = \tau_j}{s \longrightarrow \Big\{\texttt{anyOf} \mapsto \big[\{s.\texttt{enum} \mapsto [v_{i_1}, \ldots, v_{i_m}], s.\texttt{type} \mapsto \tau_i\}, \ldots, \{s.\texttt{enum} \mapsto [v_{j_1}, \ldots, v_{j_n}] s.\texttt{type} \mapsto \tau_j\}\big]\Big\}}$$

$$\text{Singleton type}\ \frac{s.\texttt{type} = [\tau_1, \ldots, \tau_n]}{s \longrightarrow \big\{\texttt{anyOf} \mapsto [\{s.\texttt{type} \mapsto \tau_1\}, \ldots, \{s.\texttt{type} \mapsto \tau_n\}]\big\}}$$

$$\text{Boolean connectives}\ \frac{c \in \mathit{JConnectives} \qquad c \in dom(s) \qquad dom(s) \setminus c \neq \emptyset}{s \longrightarrow \Big\{\texttt{allOf} \mapsto \big[\{k \mapsto v \mid k \mapsto v \in s \wedge k \in (dom(s) \setminus c)\}, \{c \mapsto s.c\}\big]\Big\}}$$

$$\text{oneOf}\ \frac{s.\texttt{oneOf} = [s_1, s_2, \ldots, s_m]}{s \longrightarrow \Big\{\texttt{anyOf} \mapsto \big[\big\{\texttt{allOf} \mapsto [s_1, \{\texttt{not} \mapsto s_2\}, \ldots, \{\texttt{not} \mapsto s_m\}]\big\}, \ldots, \big\{\texttt{allOf} \mapsto [\{\texttt{not} \mapsto s_1\}, \ldots, \{\texttt{not} \mapsto s_{m-1}\}, s_m]\big\}\big]\Big\}}$$

Figure 5. Term rewriting system for canonicalizing JSON schemas.

Jtypes from Definition 2.3, the set of relevant keywords $kw(\tau) \subset \mathit{Jkeywords}$ is given by retrieving the type-specific keywords corresponding to type τ in Table 1. Moreover, for any JSON schema s, $dom(s)$ denotes the set of keys in the key-value map s. For any type-specific keyword $k \in \mathit{Jkeywords}$, let $\mathit{default}(k)$ give the default value of k according to the JSON Schema specification. Finally, for any literal value v, let $\mathit{typeOf}(v) \in \mathit{Jtypes}$ be the JSON type of value v.

Figure 5 shows the rewriting rules for canonicalizing JSON schemas. The rules use two kind of arrows. First, $s \longrightarrow s'$ indicates a rewriting application where the lhs schema s is rewritten into the rhs schema s'. Second, $s.\mathtt{k} \mapsto v$ indicates a substitution, where the mapping of `k` is changed to the specified value v, while the mappings of all other keys of s remain unchanged.

Since schemas are normal JSON documents, they may contain keys that are not a type-specific keyword, and hence, not relevant for the validation. The first rule, *non-keywords*, drops such non-validating keys. When a schema does not have any type defined, the *missing type* rule generously assumes all JSON types are possible, which gets refined later by the *singleton type* rule.

The *homogeneous enum* rule groups enumerated values by type into a disjunction and adds the corresponding type to each member. The *singleton type* rule turns a union type, such as `{'type':[`τ_1`, `τ_2`]}`, into an explicit disjunction, such as `{'anyOf':[{'type':`τ_1`}, {'type': `τ_1`}]}`. For example, applying the singleton type and homogeneous enum rules in any order to the schema in Figure 4a yields the schema 4b.[1]

The *Boolean connectives* rule combines all Boolean connectives at the same nesting level into a conjunction, e.g., from schema 4b to 4c.

Finally, the *oneOf* rule compiles the `oneOf` into a disjunction of conjunctions.

4.2 Simplification of Intersection, Union, Negation, and Enumeration Types

After canonicalization, three Boolean connectives remain: `allOf`, `anyOf`, and `not`, which correspond to intersection, union, and negation of types. They can combine schemas of both compatible and incompatible types, e.g., strings with other strings and strings with numbers. Intersection of two incompatible types is the empty set (akin to $\bot$), and nicely enough, the schema language can express both $\top$ as the empty schema `{}` and $\bot$ as its negation `{'not':{}}`. Intersection of two compatible types involves semantically reasoning about the intersection within a known domain, e.g., string schemas intersection is decidable on classical regular expressions, and numeric schemas intersection on numeric intervals. For the intersection of Boolean connectives, we use the distributivity of intersection over union. Eventually, all intersection types are simplified either to $\bot$ or to a schema without intersections before the subtype checking. For example, the intersection type in Figure 4d yields the simplified schema in Figure 4e.

In contrast to intersection, union allows incompatible types, e.g., string or null. However, the union of compatible types requires careful reasoning to handle bounds. For instance, the union of schemas `{'maximum':5}` and `{'minimum':6}` has a gap for type `number` but is seamless for type `integer`.

Negation types are handled through De Morgan's law. For a schema s of type τ, the negation `{'not':`s`}` can be rewritten to $\{\texttt{anyOf} : [\texttt{type} : \tau_i | \tau_i \in (\mathit{Jtypes} \setminus \tau)] \cup [\bar{s}]\}$. Here, $\bar{s}$ is the complement of s, which is again computable through the semantics of the corresponding domains. For example,

[1]Note that for any schema `s`, `allOf:[s]` $\equiv$ `anyOf:[s]` $\equiv$ `s`.

Figure 4d shows how the complement for the string schema is used to eliminate the negated schema in Figure 4c.[2]

Finally, `enum` schemas are compiled into schemas without `enum` by using restrictions keywords from their corresponding JSON schema types. For instance, in Figure 4c, the enumerated empty string value is compiled into its corresponding regular expression `'^$'` before computing its complement `'.+'` in Figure 4d. This is needed for schemas with `enum` on the rhs of the subtype check, e.g., to check whether `{'minimum': 1, 'maximum':3}` <: `{'enum': [1,9]}`. The only exception is for Boolean schemas as the space of values is finite and there is no other way specify the true or false value.

Our implementation covers all cases for union and negation of *Jprimitive* types and the common cases of intersection and enumeration for all *Jtypes*, leaving uncommon cases of union, negation, and enumeration of *Jstructure* to future work. That said, this section informally describes how to handle all cases.

4.3 JSON Subschema Checking

Given two canonicalized and simplified schemas, the third step of our approach checks whether one schema is a subtype of the other. Figure 6 presents inference rules defining the subschema relation on canonical, simplified schemas. All rules are algorithmically checkable, and all rules except for *JSON-uninhabited* are type-directed. To simplify their presentation, some of the rules use unbounded quantifiers (e.g., rule *JSON-array* has an unbounded universal quantification over i and *JSON-object* over k). The text will present more complex, but equivalent, formulations that explicitly bound the computation.

The rules can be broadly divided into three groups: those for uninhabited types and `anyOf`, those for primitive types (*JSON-Boolean*, *JSON-string*, etc.), and those for structured types (*JSON-array* and *JSON-object*). We describe the rationale behind non-trivial rules in the following.

Rule *JSON-uninhabited* states that an uninhabited schema is a subtype of any other schema. It uses an auxiliary *inhabited* predicate, which is elided for space but easily computable for primitives (recall that emptiness is decidable for regular languages). For structures, the predicate ensures that the schemas of all required components are inhabited. The rule for uninhabited types is the only rule that is not type-directed. Because canonicalization generally separates schemas by type, all other rules check same-typed schemas. We can handle uninhabited schemas independently of their type because there is no actual data of that type that would require type-specific reasoning.

Rule *JSON-anyOf* handles `anyOf` schemas. Since canonicalization merges same-typed schemas, different components of an `anyOf` schema either have different types or are disjoint. As a result, it suffices to check the component schemas independently. For each schema on the left, we require a same-typed super schema on the right.

Rules *JSON-Boolean* and *JSON-null* are simple. Rule *JSON-string* is slightly more complex. Conceptually, we need to check if one regular expression is a subset of another (which is decidable) and if the left length range is included in the right length range. However, these properties are intertwined and cannot be checked independently, since the regular expression itself may limit the possible length of strings that it matches. For example, schema `{'minLength':1}` and schema `{'pattern':'.+'}` are equivalent. The former encodes the minimum length using the `minLength` field and the latter using a regular expression. To check both the regular expression and the length bound simultaneously, we encode the length bound as a regular expression, where "." matches any character, and $\{x, y\}$ allows for between x and y repetitions. If the maximum length is unbounded, we treat this as equivalent to "$.\{x, x\}.*$". We then intersect this generated pattern with the given regular expression, and use the result in our regular expression inclusion check. All of these operations are decidable for regular expressions [14].

The last primitive type, `number`, is the most complicated to handle due to `multipleOf` constraints. Canonicalization cannot push negation through `multipleOf` constraints, and it cannot combine `allOf` combinations of such negated schemas. As a result, rule *JSON-number* has to handle multiple such constraints on both sides of the relation, with or without negation. We treat simple number schemas as single-element `allOf`s for consistency. This rule verifies that any number allowed by the set of constraints on the left is also allowed by the set of constraints on the right using an auxiliary *subrange* relation, which is sketched in the following.

The *subrange* relation first normalizes all schema range bounds by rounding them to the nearest included number that satisfies its `multipleOf` constraint. For each side, it then finds the least and greatest finite bound used. Every unbounded schema is split into two (or three for totally unbounded) schemas: one (or two) that are unbounded on one side, with the least/greatest bound as the other bound. The "middle" part is bounded. All these derived schemas keep the original `multipleOf`. The bounded schemas can all be checked (exhaustively if needed). For the unbounded schemas, we can separately check the positive and negative schemas, since they do not interact in interesting ways over unbounded sets. If *PL* and *PR* are the left and right positive schemas, and *NL* and *NR* are the left and right negative schemas, we verify that the constraints divide each other:

$$\forall_{pl \in PL}, \exists_{pr \in PR},\ pl.\texttt{multipleOf} \bmod pr.\texttt{multipleOf} = 0$$
$$\forall_{nr \in NR}, \exists_{nl \in NL},\ nr.\texttt{multipleOf} \bmod nl.\texttt{multipleOf} = 0$$

The final two rules handle the two structures of JSON Schema, arrays and objects. To simplify the presentation and

[2]Simplifying negated numerical schemas with multiplicity constraint and unbounded range is not possible but deferred to the subtype checking.

$$\text{JSON-uninhabited}\ \frac{\neg\, \mathit{inhabited}(s_1)}{s_1 <: s_2} \qquad \text{JSON-anyOf}\ \frac{\forall i \in \{1..n\}, \exists j \in \{1..m\}, s_i <: t_j}{\{\texttt{anyOf} \mapsto [s_1, \ldots, s_n]\} <: \{\texttt{anyOf} \mapsto [t_1, \ldots, t_m]\}}$$

$$\text{JSON-Boolean}\ \frac{s_1.\texttt{type} = \texttt{bool} \quad s_2.\texttt{type} = \texttt{bool} \qquad s_1.\texttt{enum} \subseteq s_2.\texttt{enum}}{s_1 <: s_2} \qquad \text{JSON-null}\ \frac{\Gamma \vdash s_1.\texttt{type} = \texttt{null} \quad s_2.\texttt{type} = \texttt{null}}{s_1 <: s_2}$$

$$\text{JSON-string}\ \frac{s_1.\texttt{type} = \texttt{string} \qquad s_2.\texttt{type} = \texttt{string} \qquad s_1.\texttt{pattern} \cap \text{“}.\{s_1.\texttt{minLength}, s_1.\texttt{maxLength}\}\text{”} \subseteq s_2.\texttt{pattern} \cap \text{“}.\{s_2.\texttt{minLength}, s_2.\texttt{maxLength}\}\text{”}}{s_1 <: s_2}$$

$$\text{JSON-number}\ \frac{\begin{array}{c} s_1.\texttt{type} = \texttt{number} \wedge \ldots \wedge s_i.\texttt{type} = \texttt{number} \quad \texttt{not} \in \mathit{dom}(s_{i+1}) \wedge \ldots \wedge \texttt{not} \in \mathit{dom}(s_n) \\ t_1.\texttt{type} = \texttt{number} \wedge \ldots \wedge s_j.\texttt{type} = \texttt{number} \quad \texttt{not} \in \mathit{dom}(t_{j+1}) \wedge \ldots \wedge \texttt{not} \in \mathit{dom}(t_m) \\ \mathit{subrange}(([s_1, \ldots, s_i], [s_{i+1}, \ldots, s_n]), ([t_1, \ldots, t_j], [t_{j+1}, \ldots, t_m])) \end{array}}{\{\texttt{allOf} \mapsto [s_1, \ldots, s_i, s_{i+1}, \ldots, s_n]\} <: \{\texttt{allOf} \mapsto [t_1, \ldots, t_j, t_{j+1}, \ldots, t_m]\}}$$

$$\text{JSON-array}\ \frac{\begin{array}{c} s_1.\texttt{type} = \texttt{array} \qquad s_2.\texttt{type} = \texttt{array} \\ s_1.\texttt{minItems} \geq s_2.\texttt{minItems} \qquad s_1.\texttt{maxItems} \leq s_2.\texttt{maxItems} \\ s_2.\texttt{uniqueItems} \implies (s_1.\texttt{uniqueItems} \vee \mathit{allDisjointItems}(s_1)) \\ \forall i \geq 0 \quad s_1.\texttt{items}[i] \mid s_1.\texttt{additionalItems} <: s_2.\texttt{items}[i] \mid s_2.\texttt{additionalItems} \end{array}}{s_1 <: s_2}$$

$$\text{JSON-object}\ \frac{\begin{array}{c} s_1.\texttt{type} = \texttt{object} \qquad s_2.\texttt{type} = \texttt{object} \\ s_1.\texttt{minProperties} \geq s_2.\texttt{minProperties} \qquad s_1.\texttt{maxProperties} \leq s_2.\texttt{maxProperties} \\ s_1.\texttt{required} \supseteq s_2.\texttt{required} \\ \forall k, \quad \{\texttt{allOf} \mapsto \{s_1.\texttt{properties}.k\} \cup s_1.\texttt{patternProperties}.\langle k \rangle\ \} \mid s_1.\texttt{additionalProperties} \\ <: \ \{\texttt{allOf} \mapsto \{s_2.\texttt{properties}.k\} \cup s_2.\texttt{patternProperties}.\langle k \rangle\ \} \mid s_2.\texttt{additionalProperties} \\ \forall k, \quad \{k\} \cup s_1.\texttt{dependencies}.k \subseteq \mathit{dom}_{obj}(s_1) \implies \{k\} \cup s_1.\texttt{dependencies}.k \subseteq \mathit{dom}_{obj}(s_2) \\ \vee\ \{\texttt{allOf} \mapsto [s_1, s_1.\texttt{dependencies}.k]\} <: \{\texttt{allOf} \mapsto [s_2, s_2.\texttt{dependencies}.k]\} \end{array}}{s_1 <: s_2}$$

Figure 6. JSON Schema subtype inference rules.

to unify multiple cases, we introduce a *default* operator. If a is defined, then $a \mid b$ evaluates to a, otherwise it returns b.

The *JSON-array* rule checks two arrays. The lhs array size bounds should be within the size bounds of the rhs array. Additionally, the schema of every item specified in the former needs to be a subschema of the corresponding specification in the latter. If a schema is not explicitly provided, the schema provided by `additionalItems` is used. Recall that canonicalization adds in a default `additionalItems` schema if it was not specified.

Additionally, if the right side specifies that the items must be unique, then the left needs to either specify the same, or implicitly enforce this. As a motivating example,

$$\{\texttt{items} \mapsto [\{\texttt{enum} \mapsto [0]\}, \{\texttt{enum} \mapsto [1]\}]\}$$

is a subschema of $\{\texttt{uniqueItems} \mapsto \texttt{true}\}$.

The *allDisjointItems* predicate checks for this by first obtaining the set of all the effective item schemas: every item schema for an index within the specified min/max bounds, and `additionalItems` if any allowed indices are unspecified. It then verifies that the conjunction of all pairs of effective items schemas are uninhabited.

The rule for *JSON-array* quantifies over non-negative indices i, which, in principle, is a problem. However, it suffices to check for i up to max(length(s_1.items), length(s_2.items)). This check ensures that all specified item schemas are checked, along with the two `additionalItems` schemas.

The *JSON-object* rule checks two object schemas. We first verify that the number of properties of both sides have the appropriate relation, and that the left side requires all the keys that the right side requires. Next, for every key, we first determine which schema is appropriate for that key. If a key is in `properties` or matches (one or more) regular expressions in `patternProperties`, we take the conjunction of all those schemas using *allOf*. Otherwise, we use the schema specified in `additionalProperties`. The derived schemas for the key are then compared. Finally, for every key, we ensure that the dynamic schemas induced by dependencies[3] have the appropriate relationship.

[3]Dependencies allow the schema of an object to change based on the presence of certain properties.

The rule for checking objects quantifies over the unbounded set of all keys, inhibiting computation. For checking dependencies, resolving this is simple, since we need only check the finite set of keys in the domain of the dependency map of s_1. For properties, it is a bit more complex. First, note that we can treat properties as pattern properties with a constant pattern. Second, we can treat additional properties as a pattern property whose key is the negation of the union of all the other patterns. Finally, we can remove any patterns that are uninhabited.

What remains is to check the two derived pattern property maps. It is simple to check that each schema associated with a pattern on the left is a subschema of all the schemas on the right that intersect it. However, while this would be sound, it would be overly restrictive. For example, the object schema

$$\{\text{“a|b”} \mapsto \{\texttt{pattern} \mapsto \text{“x*y*”}\},\ \text{“a|c”} \mapsto \{\texttt{pattern} \mapsto \text{“x*z*”}\},\ \text{“b|c”} \mapsto \{\texttt{pattern} \mapsto \text{“y*z*”}\}\}$$

is, according to the *JSON-object* rule, a subschema of

$$\{\text{“a|b|c”} \mapsto \{\texttt{pattern} \mapsto \text{“x*|y*|z*”}\}\}$$

but would fail this simple test. Every key matches a (different) set of overlapping patterns, and their combined schemas each satisfy the subschema relation.

To check more exactly, we need a more complicated check. First, we break up the patterns on the left side into disjoint groups (the unions of each group are disjoint).[4] For each group, we proceed as follows: If there are n patterns and schemas in the group, we first take the intersection of all n patterns (and their corresponding schemas). If the resulting pattern intersection is non-empty, then we find all the schemas on the right that overlap with the derived pattern (their intersection is non-empty), and check the subschema relation for them. Next, we find all intersections of $n-1$ patterns, and subtract the intersection of all n patterns. For each of these intersection, we find the overlapping patterns on the right and again check the subschema relation. Similarly, for all intersection of all $n-2$ patterns, subtracting out all the intersections of size n and $n-1$ patterns, and so on. This procedure requires a bounded number of regular expression operations and subschema checks, and so can be computed. For simple objects with no pattern properties, all keys become their own disjoint group, and the test degenerates into a simple test of corresponding schemas.

5 Implementation

We implemented our subschema checker as a Python tool in around 1,300 lines of code. The implementation builds upon the jsonschema library[5] to validate schemas before running our subtype checking, the greenery library[6] for computing intersections of regular expressions, and the jsonref library[7] for resolving JSON schemas references.

[4]This grouping is for performance and not required for correctness.
[5]https://github.com/Julian/jsonschema
[6]https://github.com/qntm/greenery

6 Evaluation

This section evaluates the implementation of our JSON subschema checker, which we refer to as *jsonsubschema*. It answers the following research questions:

RQ$_1$ How correct is *jsonsubschema* in practice?
RQ$_2$ How does *jsonsubschema* fare against existing work?
RQ$_3$ How complete is *jsonsubschema* in practice?
RQ$_4$ How efficient is *jsonsubschema*?

6.1 Experimental Setup

We evaluate our subschema checker on three datasets of JSON schemas from different domains: *WP*, *K8s*, and *Lale*. *WP* is a collection of schemas describing content used by the Washington Post within the *Arc Publishing* content creation and management system.[8] *K8s* is the set of JSON schemas describing the OpenAPI specifications for *Kubernetes*, an open-source system for automating deployment, scaling, and management of containerized applications.[9] Since OpenAPI specifications contain more information beyond the schemas for the REST API endpoints, we use a set of JSON schemas extracted from them.[10] Specifically, we used the standalone flavor of schemas where `$ref` have been resolved to local files. The *Lale* dataset is a set of schema pairs from the LALE open-source project[11], which is a Python library for type-driven automated machine learning.

The first two datasets, *WP* and *K8s*, comprise several versions. We apply our subschema checker across each pair of consecutive versions of the same schema that introduces some textual modification, to spot whether the change may impact the compatibility of the corresponding systems. We use the third dataset, *Lale*, to find type errors in AI pipelines where wrong operators could be applied to specific datasets. We consider 4 *Lale* operators and 7 datasets, yielding 28 schema pairs.

Table 2 shows statistics for each dataset. For example, *K8s* has in total 124 versions, with a total of 86,461 schemas. Due to the additions and deletions of schemas between pairs of consecutive versions, the total number of pairs of schemas across all pairs of subsequent versions is 82,814. Finally, since not every new version of an API modifies every schema, we only keep pairs of non-equal files. Overall, the total number of pairs of schemas is 8,548. Many of the schemas are of non-trivial size, with an average of 56KB and a maximum of 1,047KB.

[7]https://github.com/gazpachoking/jsonref
[8]https://github.com/washingtonpost/ans-schema
[9]https://kubernetes.io/
[10]https://kubernetesjsonschema.dev/
[11]https://github.com/ibm/lale

Table 2. Dataset details.

Dataset	Versions	Schemas	Pairs	Modified pairs
WP	28	2,604	2,411	2,060
K8s	124	86,461	82,814	6,460
Lale	–	–	28	28
Total				8,548

All experiments are performed on an Intel Core i7-4600U CPU (2.10GHz) machine with 16GB of memory running Ubuntu 18.04 (64-bit).

6.2 Correctness in Practice

For RQ_1, we evaluate the correctness of both components of our approach: canonicalization and subtype checking.

6.2.1 Canonicalization

The canonicalization aims at producing a valid canonical schema that is semantically equivalent to the input schema.

To test validity, *jsonsubschema* checks the canonicalized schema against the meta-schema of JSON Schema using an off-the-shelf JSON schema validator (Section 5). Across our entire dataset, there is no single case where this check fails.

To check that canonicalization preserves the semantics of the original input schema, we apply the canonicalization component on the official test suite for JSON Schema draft-04.[12] This widely-used test suite for JSON Schema validators provides 146 schemas and 531 tests that offer full coverage of the JSON Schema specification. Each test provides a JSON document d to be validated against a specific schema s and the expected correct validation behavior *res*, i.e., tests are of the form $\text{JValid}(d, s) = \mathit{res}$. We tested whether $\forall s, \forall d, \text{JValid}(d, s) = \mathit{res} \implies \text{JValid}(d, \text{canonical}(s)) = \mathit{res}$.

Our canonicalizer successfully canonicalizes 129 out of the 146 schemas. The cases where canonicaliztion fails are similar to what we discuss in 6.4. When canonicalization fails, we cannot run the corresponding validation tests. Therefore, canonicalized schemas pass 457 out of the 473 tests the validator passes on un-canonicalized test schemas.

These results, of course, show only that canonicalization in most of the cases does not yield an invalid or a more strict schema than the input schema. The following experiments on the correctness of subtype checking rely on correct canonicalization, and hence, provide additional evidence.

6.2.2 Subtype Checking

Self-comparison. As an automated, large-scale correctness check of the subtype checking, we perform a simple sanity check that asks *jsonsubschema* whether a schema is equivalent to itself (Definition 3.2). We randomly sample 4,200 schemas and run our subschema checker using the same schema on both sides of the subtype relation, i.e., checking for a schema s whether $s <: s$. Our subtype checking does not rely on any sort of structural equality. Therefore, in this setup, our implementation is oblivious to the fact that both schemas are the same, so it canonicalizes both schemas and then performs subtype checking. In all 4,200 samples, this test passes correctly.

[12]https://github.com/json-schema-org/JSON-Schema-Test-Suite

Table 3. Effectiveness of *jsonsubschema* and comparison to existing work.

		jsonsubschema					*issubset*				
	Pairs	**Fail**	**TP**	**TN**	**FP**	**FN**	**Fail**	**TP**	**TN**	**FP**	**FN**
$<:$	35	0	29	6	0	0	10	9	0	6	10
$:>$	35	0	31	4	0	0	10	21	0	4	0
$\equiv$	100	0	100	0	0	0	50	27	0	0	23
$\not\equiv$	100	0	63	37	0	0	0	63	0	37	0
Lale	28	0	12	16	0	0	7	3	10	0	8
Total	298	0	235	63	0	0	77	123	10	47	41

Comparison against a ground truth. For further validation, we compare the results of *jsonsubschema* against a ground truth. Specifically, we gather pairs of schemas, along with their expected subtype relationship, in three ways. First, we randomly sample pairs that are textually different and manually assess their subtype relationship. Second, we sample consecutive versions of schemas from *WP* and *K8s* and manually assess their subtype relationship. For example, we consider versions 0.7.0 and 0.7.1 of the `utils/named_entity` schema from the *WP* dataset. This sample of pairs represents the usage scenario where our approach checks whether an evolving API specified through a JSON schema may break an application. Third, for the 28 schema pairs from *Lale*, the ground truth is whether or not the corresponding machine-learning operator throws an exception when training on the corresponding dataset. By checking the schemas statically, our subtype checker can avoid such runtime errors.

In total, we gather 298 pairs with a ground truth subtype relationship, as summarized in Table 3. The $<:$, $:>$, $\equiv$, and $\not\equiv$ symbols represent what test we performed on each pair. For example, for each pair $\langle s, t\rangle$ in the $<:$ row, the ground truth indicates whether $s <: t$ holds (positive, P) or not (negative, N). The *jsonsubschema* part of the table shows the results of applying our subschema checker to each pair. The TP, TN, FP, and FN columns indicate the true positives, true negatives, false positives, and false negatives, respectively. For example, TN means that the tool produces the correct result (T for true) and that the ground truth indicates that the relationship being tested is not expected to hold (N for negative). Our tool produces the correct results for all 298 schema pairs in the ground truth.

6.3 Comparison to Existing Work

As we discuss in Section 7, our work is the first to formally define the subschema relation on JSON schemas and present an algorithm to perform this check. Therefore, to our knowledge, there is no direct formal work that we can compare against. However, since our work is motivated by the practical need for subtyping JSON schemas, for RQ_2, we compare our tool to the closest developer-tool we could find, is-json-schema-subset (*issubset*) [12]. The *issubset* tool is written in TypeScript and its documentation states the same goal as ours: "Given a schema defining the output of some process A, and a second schema defining the input of some process B, will the output from A be valid input for process B?" We use the most recent version, which is 1.0.6.

We first ran *issubset* on all three datasets, *WP*, *K8s*, and *Lale*. It failed to run on some schemas from *WP* and *Lale* due to unsupported schema version, although the *issubset* documentation does not describe such a limitation. Next, we compare correctness results for *issubset* against our *jsonsubschema*. The right part of Table 3 shows the correctness results for *issubset* using the methodology described in Section 6.2.2.

The first observation is that *issubset* produces a non-negligible number of true positives, which means it indeed captures some of the semantic of the subtyping relation. However, we notice that its false positive and false negative rates are also non-negligible. For instance, on detecting <:, it has 60% precision and 47% recall, whereas our approach has 100% precision and recall.

To get a better understanding of the low recall, we inspected the code of *issubset* and tested it on simple handcrafted schemas. We find that although the tool performs some simple semantic checks, e.g., it correctly reports `{'type':'integer'}` <: `{'type':'number'}`, it lacks the ability to capture the richness of JSON schema in many ways. For instance, it fails to detect `{'type':['string','null']}` ≡ `{'type':['null','string']}`, and is oblivious to uninhabited schemas, such as `{'type':'string','enum':[1]}`.

6.4 Completeness in Practice

Being complete in practice is difficult. To balance completeness and effort, there is a set of features our approach currently cannot deal with. Therefore, for RQ_3, we report on failure cases of *jsonsubschema*, i.e, cases where we do not produce a subtype decision, either due to an unimplemented feature or due to a limitation of the approach.

Table 4 show the cases when *jsonsubschema* fails. In total, out of 8,548 schemas pairs, the subschema checks failed for 5.85% of cases. The upper part of the table shows the three limitations due to our approach or implementation. The first and most dominant failure reason is circular and recursive schemas. Although our approach currently does not handle recursive schemas, we know theoretically that subtyping recursive types is decidable [1]. The second case is the rare use case of negating objects schemas. We opted not to implement this negation of arrays and objects believing that they are rarely used. Indeed, as seen in Table 4, only 0.34% of schema pairs fail due to the absence of this feature. The third case is when string schemas or object schemas use non-classical regular expressions (e.g., with positive and negative look-around). This limitation is beyond our scope since non-classical regular expressions are not regular languages.

Table 4. Reasons for incompleteness.

Failure reason	**Count**	**%**
Unimplemented feature		
Recursive or circular `$ref`	453	5.30%
Negated object schema	29	0.34%
Non-regular regex pattern	5	0.06%
Other errors		
Unresolved refs	11	0.13%
Invalid JSON file	2	0.02%
Total	500	5.85%

The lower part of Table 4 shows failure cases orthogonal to our approach. For eleven pairs of schemas, at least one of the schemas had an invalid `$ref`. For two pairs of schemas, at least one of the two files is not a valid JSON document.

6.5 Efficiency

To evaluate how fast our subschema checker is in practice for RQ_4, we measure the time taken by subschema checks on a sample of 798 pairs of non-equal schemas from Table 2. We took every time measurement ten times and we report the average. Figure 7 shows the size of pairs of schema files in KB against the time subschema checking takes in seconds.

In most cases, our subschema checker terminates within a few seconds for moderately sized schemas, with time increasing roughly linearly with the schema file size. However, our subschema approach is lazy and terminates on the first violation of a subtyping rule. On one pair of schemas in our dataset, eliminated from the figure for scaling sake, it took around 2.8 minutes to terminate, which is not optimal for production. We will explore how to improve on this, for instance, by on-demand canonicalization.

7 Related Work

7.1 JSON Schema and Schema Subtyping

Practitioners have significant interest in reasoning about the subtype relation of JSON schemas. Section 6.3 has an experimental comparison against the strongest competitor among the available tools, is-json-schema-subset [12], which was developed concurrently with our work. Another closely

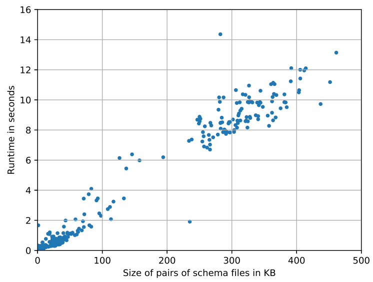

Figure 7. Efficiency of Subschema checking.

related tool[13] relies on simple syntactic checks. For example, that work considers a change as a breaking change whenever a node is removed from the schema. As illustrated by Figure 4, removing nodes (and replacing them by others) may yield not only subtypes but even equivalent schemas. Yet another existing tool[14] checks whether two schemas are equivalent but does not address the subtyping problem.

[19] formally define the syntax and semantics of JSON Schema, including the JSON validation problem. An alternative formulation of JSON validation uses a logical formalism [6]. [2] address the problem of inferring schemas for irregular JSON data, but their work does not use the JSON Schema standard we are targeting here. None of the above pieces of work addresses the subschema problem.

There are other schema definition languages for JSON besides JSON Schema. One popular alternative is the Swagger/OpenAPI specification language.[15] While similar to JSON Schema, it is not fully compatible. The swagger-diff tool[16] aims at finding breaking API changes through a set of syntactic checks, but does not provide the detailed checks that we do. Avro is another schema definition language[17], which, however, does not specify a subschema relation. We envision our work to help define subtype relations of these alternative schema definition languages.

7.2 Type Systems for XML, TypeScript, and Python

CDuce is a functional language designed for working with XML, which can reason about the types of XML documents and about subtype relations [4]. [24] address the problem of subschema checking for XML, where it is called schema containment. These approaches treat XML as tree automata, which is impossible for JSON, as JSON Schema is more expressive than tree automata [19].

Both JavaScript and Python have a convenient built-in syntax for JSON documents. Furthermore, there are type systems retrofitted onto both JavaScript [5] and Python [25]. Therefore, a reasonable question to ask is whether JSON schema subtype queries could be decided by expressing JSON documents in those languages and then using the subtype checker of those type systems. Unfortunately, this is not the case, since JSON Schema contains several features that those type systems cannot express. For instance, JSON Schema supports negation, `multipleOf` on numbers, and `pattern` on strings, none of which those type systems support.

7.3 Applications of Subschemas

One application of JSON subschema is for statically reasoning about breaking changes of web APIs. A study of the evolution of such APIs shows that breaking changes are frequent [16]. Another study reports that breaking changes of web APIs cause distress among developers [9]. Since JSON schemas and related specifications are widely used to specify data types, our subschema checker could help identify breaking changes already statically and on the schema-level, instead of relying on testing.

Data validation for industry-deployed machine learning pipelines is of crucial value as such pipelines are usually retrained frequently with new data. In order to validate incoming data, Google TFX [3] synthesizes a custom data schema based on statistics from available data and uses this schema to validate future data instances fed to the TensorFlow pipeline [7]. Amazon production ML pipelines [22] offer a declarative API which allow users to manually define desired constraints or properties of data. Then data quality metrics such as completeness and consistency are measured on real-time data with respect to the pre-defined constraints and anomalies are reported. Both systems are missing an explicit notion of schema subtyping. For instance, TFX uses versioned schemas to track the evolution of inferred data schemas, and reports back to the user whether to update to a more (or less) permissive schema based on the historical and new data instances [3]. Lale uses JSON schemas to specify both correct ML pipelines and their search space of hyperparameters [13]. The ML Bazaar also specifies ML primitives via JSON [23]. Another type-based system for building ML pipelines is described by [20]. These systems could benefit from JSON subschema checking to avoid running and deploying incompatible ML pipelines.

8 Conclusion

This paper introduces a subtype checker for JSON Schema. There are several features in JSON Schema that make subtype checking difficult, including a full set of Boolean connectives, enumerations containing values of possibly heterogeneous other types, regular expressions for strings, and multiple-of constraints for numbers. Our checker is the first to effectively handle these cases. The evaluation demonstrates that the tool works well on a large set of examples of high real-world importance, including web APIs, cloud computing, and artificial intelligence.

[13] https://bitbucket.org/atlassian/json-schema-diff-validator
[14] https://github.com/mokkabonna/json-schema-compare
[15] https://swagger.io/
[16] https://github.com/civisanalytics/swagger-diff
[17] http://avro.apache.org/

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